CS 21 Decidability and Tractability **Winter 2008**

Problem Set 4

Out: February 13 *Due:* **February 20**

Reminder: you are encouraged to work in groups of two or three; however you must turn in your own write-up and note with whom you worked. You may consult the course notes and text (Sipser). Please attempt all problems. **To facilitate grading, please turn in each problem on a separate sheet of paper and put your name on each sheet. Do not staple the separate sheets.**

1. State whether each of the following statements is true or false, and briefly justify your answer. In this problem the reductions are always polynomial time many-one (or "mapping") reductions.

   (a) It is possible that every language in EXP has algorithms that run in time $O(2^{n^{100}})$.

   (b) Either P $\neq$ NP or NP $\neq$ EXP (or both).

   (c) If for some fixed k, an NP-complete language has an $O(n^k)$-time algorithm, then every language in NP has an $O(n^k)$-time algorithm.

   (d) If P = NP, then every language in NP is NP-complete.

   (e) Every language in NP reduces to every EXP-complete language.

2. A graph G is called *k-colorable* if there is a way to assign a color to each vertex so that no edge has both endpoints assigned the same color, using at most k distinct colors. Show that the language

   $$\text{2-COLORABLE} = \{G : G \text{ is 2-colorable}\}$$

   is in P by reducing it to a problem known to be in P.

3. Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are *isomorphic* if there is a bijection $f : V_1 \to V_2$ such that $(u, v) \in E_1 \Leftrightarrow (f(u), f(v)) \in E_2$. For a given graph H, define the following language:

   $$\text{CONTAINS}_H = \{G : G \text{ contains a subgraph isomorphic to } H\}.$$

   Here by "subgraph" we mean a subset of G's vertices together with all of G's edges on that subset of vertices – often called an "induced subgraph." Prove that for every H, CONTAINS$_H$ is in P.

4. Show that the following problem is in P:

   $$\text{UNARY SUBSET SUM} = \left\{(1^B, x_1, x_2, \ldots, x_n) : \exists \text{ a multiset } I \text{ of } [n] \text{ for which } \sum_{i \in I} x_i = B\right\}.$$

   Here the x_i are all positive integers, as is B, and $[n]$ is shorthand for the set $\{1, 2, 3, \ldots, n\}$. The notation 1^B means a string of B ones, which is the representation of B in unary. Hint: solve the problem for all $B' \leq B$.