CS 153 Current Topics in Theoretical Computer Science **Spring 2022**

Course Summary and Syllabus

Lecturer: Chris Umans *Date: March 29*

Course summary:

We'll spend the term studying matrix multiplication, the most prominent open "upper bounds" problem in the Algebraic Complexity. We'll touch on the key results since Strassen's algorithm, and develop the so-called "group-theoretic" approach in detail. Along the way we'll study several conjectures that would imply "exponent 2" algorithms for matrix multiplication, and develop methods for ruling out certain families of groups as potential routes to exponent 2 within this framework.

Course Information:

- Instructor: Chris Umans (`umans@cs.caltech.edu`)
- Lectures: Tuesdays and Thursdays 1:00 – 2:25 in Annenberg 213
- Office hours: TBD
- Text: None. Some preliminary material in the course is covered/surveyed in these documents:
  - Notes from the course "Complexity of Bilinear Problems." Markus Bläser. 2009. Available at `Complexity of Bilinear Problems.pdf`

  After developing the requisite background, we'll work through the key results in a sequence of papers. Tentatively, these will be:

  - H. Cohn and C. Umans. "A Group-theoretic Approach to Fast Matrix Multiplication." 2003. Available at `https://arxiv.org/abs/math/0307321`
  - H. Cohn, R. Kleinberg, B. Szegedy, and C. Umans. "Group-theoretic Algorithms for Matrix Multiplication." 2005. Available at `http://users.cms.caltech.edu/~umans/papers/CKSU05.pdf`
  - H. Cohn and C. Umans. "Fast Matrix Multiplication using Coherent Configurations." 2013. Available at `http://www.cs.caltech.edu/~umans/papers/CU12.pdf`
  - N. Alon, A. Shpilka, and C. Umans. "On Sunflowers and Matrix Multiplication." 2013. Available at `https://link.springer.com/article/10.1007/s00037-013-0060-1`
  - J. Blasiak, T. Church, H. Cohn, J. Grochow, E. Naslund, W. Sawin, and C. Umans. "On cap sets and the group-theoretic approach to matrix multiplication". 2017. Available at `https://discreteanalysisjournal.com/article/1245-on-cap-sets...`
  - J. Blasiak, T. Church, H. Cohn, J. Grochow and C. Umans. "Which groups are amenable to proving exponent two for matrix multiplication?" 2017. Available at `https://arxiv.org/abs/1712.02302`
  - J. Blasiak, H. Cohn, J. Grochow, K. Pratt, and C. Umans. "Matrix Multiplication via Matrix Groups." 2022. Available at `TBA`
- Webpage: `http://www.cs.caltech.edu/~umans/cs153/`

Prerequisite: This course is pitched at a beginning graduate level, but both undergrads and grad students are encouraged to attend. Prerequisites are mathematical maturity and curiosity. The course is intended to be largely self-contained, but exposure to elementary probability and algebra, as well as material covered in CS21, CS38 and CS151, is helpful.

Course requirements:

Course participants should attend/participate in lectures (20%), complete 2 short problem sets (30%), and read and present a relevant research paper at the end of the term (50%).

Possible topics/papers for presentation will be listed on the webpage.

Honor code: For problem sets, you may consult *only* the following material: (1) lecture slides and problem sets posted on the class webpage, (2) solution sets for problem sets you have already turned in, (3) course notes you or others took during lecture, and (4) and of the referenced texts and papers. **I am aware there may be material from past iterations of this course readily available (online and elsewhere). Apart from the aforementioned sources, you may not seek out, study from, or otherwise consult this material during the term, starting now (March 29, 2022).** Please feel free to ask me for clarification if any of these guidelines are unclear.

Collaboration policy: Collaboration on problem sets is encouraged, and you may work together in small groups to figure out a solution, including working out details of parts that are challenging or may require a clever trick. You must however turn in your own writeup that may draw on *ideas* from your group, even in detail, but *you may not use or look at the completed work of others.* The writeup should note with whom you worked. You are individually responsible for learning and understanding the course material in preparation for the exams, and this is not likely to happen if you rely too heavily on your collaborators!