

## Course Summary and Syllabus

*Lecturer: Chris Umans**Date: April 3***Course summary:**

This year's focus will be Algebraic Complexity.

We'll spend roughly half of the term studying matrix multiplication, the most prominent open "upper bounds" problem in the field. We'll cover the key results since Strassen's algorithm, many of which play a role in the recent breakthrough of Stothers (2010) and Williams (2011). We'll also study several conjectures that would imply "exponent 2" algorithms for matrix multiplication.

The second half of the term will be devoted to lower bounds. We'll frame Valiant's algebraic analogue of the P vs. NP question, and cover lower bounds in restricted models: multilinear formulas, bounded-depth circuits, bounded-coefficient computations, and others.

**Course Information:**

- Instructor: Chris Umans ([umans@cs.caltech.edu](mailto:umans@cs.caltech.edu))
- Lectures: Tuesdays and Thursdays 1:00 – 2:25 in Annenberg 314
- Office hours: TBD
- Text: None. Significant portions of the material in the course is covered/surveyed in these three documents, available online:
  - Notes from the course "Complexity of Bilinear Problems." Markus Bläser. 2009. Available at <http://www-cc.cs.uni-saarland.de/teaching/SS09/ComplexityofBilinearProblems/script.pdf>
  - "On the Complexity of Matrix Multiplication" (Ph.D. thesis). Andrew Stothers. 2010. Available at <http://www.maths.ed.ac.uk/pg/thesis/stothers.pdf>
  - "Arithmetic circuits: a survey of recent results and open questions." Amir Shpilka and Amir Yehudayoff. 2010. Available at <http://www.cs.technion.ac.il/~shpilka/publications/SY10.pdf>

The webpage will be updated regularly with links to relevant papers.

- Webpage: <http://www.cs.caltech.edu/~umans/cs153/>

**Prerequisite:** This course is pitched at a beginning graduate level, but both undergrads and grad students are encouraged to attend. Prerequisites are mathematical maturity and curiosity. The course is intended to be largely self-contained, but exposure to elementary probability and algebra, as well as material covered in CS21, CS38 and CS151, is helpful.

**Course requirements:**

Course participants should attend/participate in lectures (20%), complete 2-3 short problem sets (20%), and read and present a relevant research paper at the end of the term (60%).

(Very) tentative lecture schedule:

Lecture	Date	Topic
1	Apr 3	Introduction; models of algebraic computation, Strassen's algorithm
2	Apr 5	border rank, Asymptotic Sum Inequality
3	Apr 10	group-theoretic approach
4	Apr 12	group-theoretic approach
5	Apr 17	group-theoretic approach
6	Apr 19	Coppersmith-Winograd bounds
7	Apr 24	Coppersmith-Winograd bounds
8	Apr 26	reductions between linear algebra problems
9	Apr 27	matrix multiplication conjectures and sunflower conjectures
10	Apr 29	catch up
11	May 1	lower bounds preliminaries, Valiant's P, NP and permanent vs. determinant
12	May 3	lower bounds via degree, monotone and non-commutative computation
13	May 8	constant-depth
14	May 10	constant-depth
15	May 15	multilinear
16	May 17	multilinear
17	May 22	bounded coefficient
18	May 24	catch up
19	May 29	paper presentations
20	May 31	paper presentations

Possible topics/papers for presentation will be listed on the webpage.