Phase transitions in large graphical models: from physics to information theory and computer science

Andrea Montanari

Stanford University

March 15, 2007

Outline

1. An instructive story and many questions
2. The general theme: Phase transitions and Graphical models
3. A couple of applications (for time limits)
   - Modern coding theory
   - Random constraint satisfaction problems

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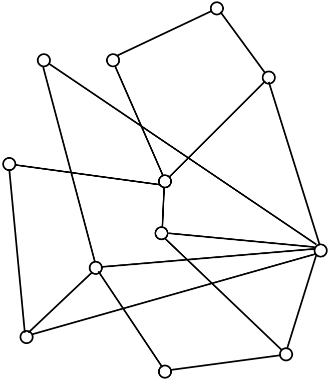
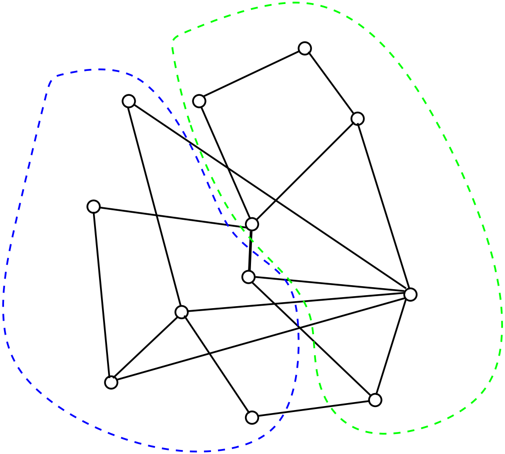
An instructive story and many questions

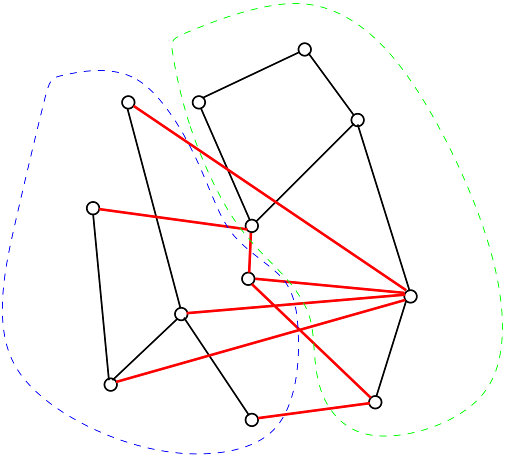
Given a graph. . .



. . . we want to partition its vertices . . .

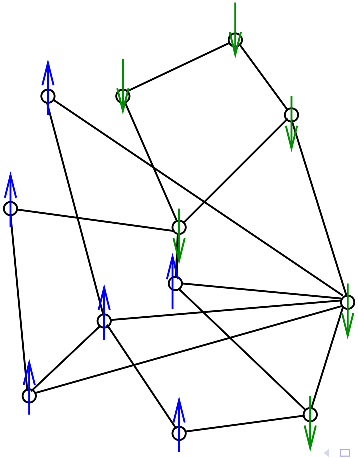


. . . to maximize the number of edges across.



The physics version

Localized magnetic moments (spins)
Antiferromagnetic interaction (graph)



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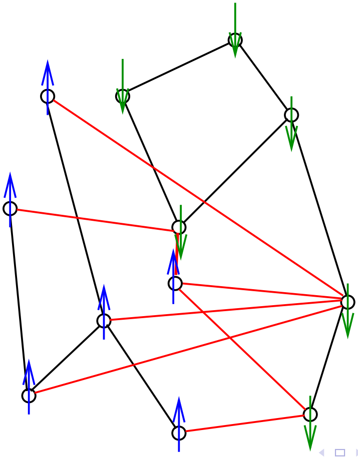

CS LENS

MAXCUT

NP-hard to approximate

A few questions I like

- What is the structure of *low energy configurations/optimal cuts*?
- How does Nature *find the optimum*? How would we find it?
- Is there a *'physics theory'* to describe low energy configurations?
  Is there an *'efficient algorithm'* to find optimal cuts?

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Start with 'simple' model

Connect each pair of vertices with probability 0.5 (independently)

A random partition yields

$$|\mathrm{CUT}| \approx \frac{1}{2}|\mathrm{EDGES}|\,.$$

SK (1972): How better is the optimal partition?

$$|\mathrm{CUT}| = \frac{1}{2}|\mathrm{EDGES}| + \frac{1}{4}\Delta\,|\mathrm{NODES}|^{3/2} + \cdots$$

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Where

$$\begin{aligned}
\Delta &= \frac{1}{4}\inf_q \left\{ \int_0^\infty (1-q^2(x)) - \phi_q(0,0) \right\} \\
\frac{\partial \phi(y;x)}{\partial x} &= -\frac{1}{2} q'(x) \left[\frac{\partial^2 \phi(y;x)}{\partial y^2} + x \left(\frac{\partial \phi(y;x)}{\partial y} \right)^2 \right] \\
\phi(y;\infty) &= |y|
\end{aligned}$$

Conjecture : Parisi (1979)

Proof : Guerra, Talagrand (2004)

CS LENS

$\Delta = \inf_q \mathcal{F}[q]$
Is there any hidden duality in the problem?

Flipping (spins 1 and 2) $\approx$ Flipping (1)+ Flipping (2)
Can this fact be exploited algorithmically?

Physical dynamics is 'local'
How do local optimization algorithms work?

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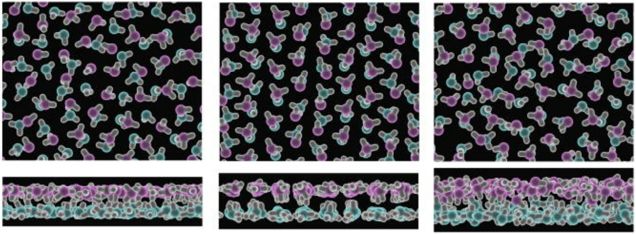
Phase transitions and Graphical models

What is a phase transition?

`An` `abrupt change` `in the state of a 'large' system as some control parameter is varied.`

Example: water is liquid at 0.01°C and solid at -0.01°C.

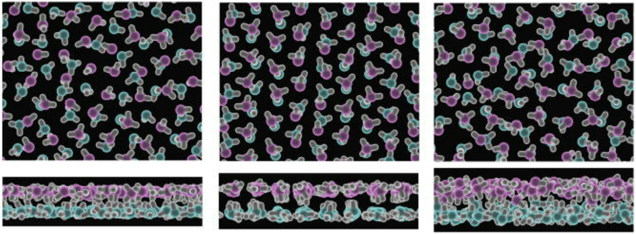


a. $H = 0.58$ nm b. $H = 0.66$ nm c. $H = 0.74$ nm

What is a phase transition?

A phase transition is accompanied by the emergenge of long range correlations.

Example: water is liquid at $0.01°$C and solid at $-0.01°$C.



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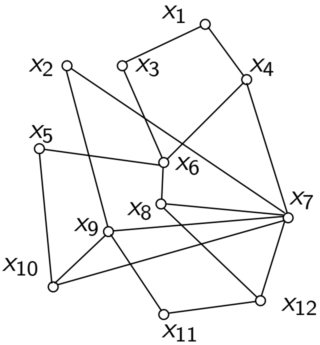
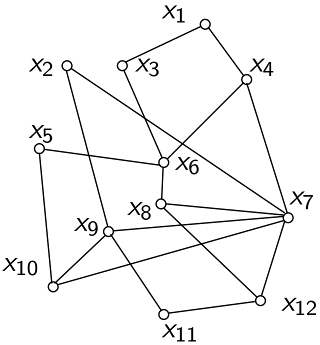
Probability+Locality



$$\mu(\underline{x}) = \frac{1}{Z} \prod_{(ij)\in E} \psi_{ij}(x_i, x_j)\,, \qquad \underline{x} = (x_1, \dots, x_n).$$

(statistical physics, counting, inference, estimation, coding,. . .)

MAXCUT – Antiferromagnet



$$\mu(\underline{x}) = \frac{1}{Z} \prod_{(ij)\in E} \exp\{-\beta x_i x_j\}\,, \qquad x_i \in \{+1,-1\}\,.$$

Generic computational tasks

- Optimization

$$\underline{x}_* = \arg\max \prod_{(ij)\in E} \psi_{ij}(x_i, x_j)\,.$$

- Partition function

$$Z = \sum_{\underline{x}} \prod_{(ij)\in E} \psi_{ij}(x_i, x_j)\,.$$

- Marginals

$$\mu(x_i) = \sum_{x_{\sim i}} \mu(\underline{x})\,.$$

- Sampling.

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CS LENS

Are far apart variables/particles strongly correlated?
Can we approximate marginals $\mu(x_i)$ using only local information?

Can the system be found in different phases?
What is the 'qualitative' structure of $\mu(\,\cdot\,)$?
(conductance/concentration)

Does it relax rapidly to equilibrium?
Can we sample/optimize with local algorithms?

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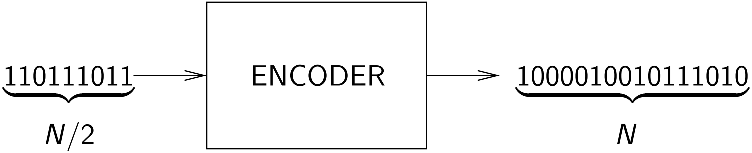
A couple of applications

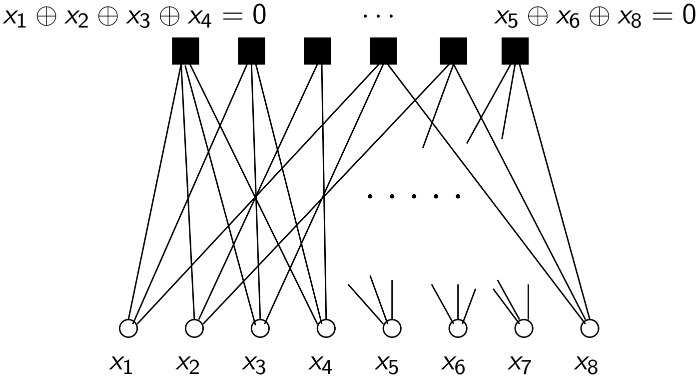
Modern coding theory

To be concrete:
coding over binary memoryless symmetric channels.

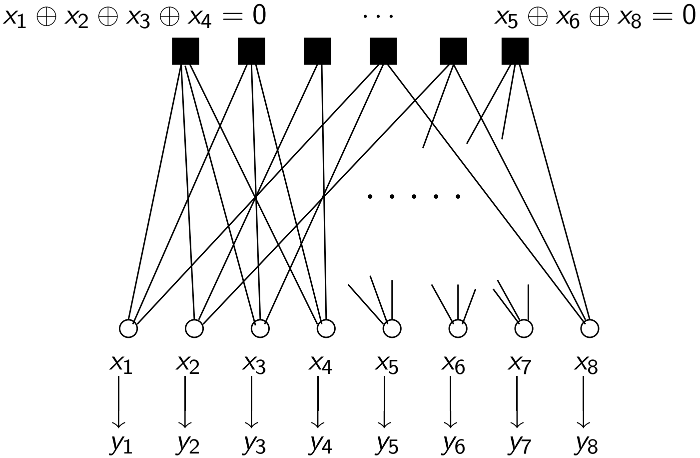


encoder $\Leftrightarrow$ constraints over message bits

LDPC codes [Gallager 1963, MacKay 1995]



constraints over message bits $\Leftrightarrow$ graphical representation



$$\mu(x|y) = \frac{1}{Z(y)} \mathbb{I}(x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0) \cdots \mathbb{I}(x_5 \oplus x_6 \oplus x_8 = 0) \cdot$$

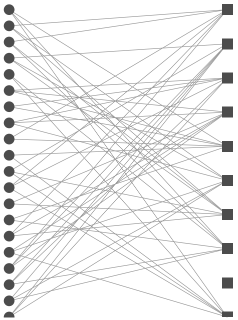
$$\cdot\, Q(y_1|x_1) \cdots Q(y_8|x_8)$$

From 10^2 to 10^5 message bits

Random graph

Iterative message passing decoding

Message Passing + Density evolution analysis



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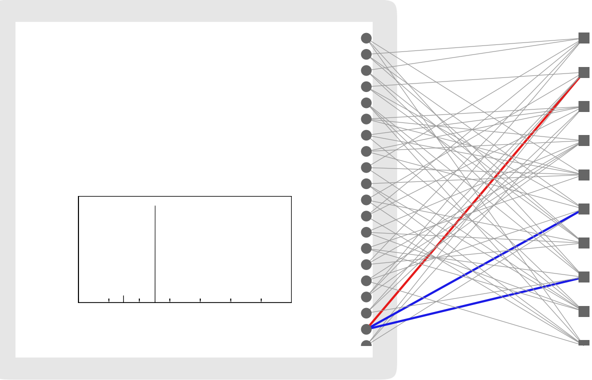

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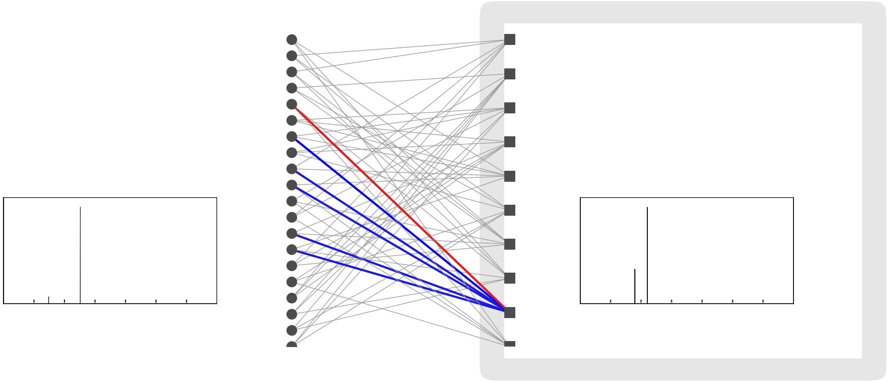

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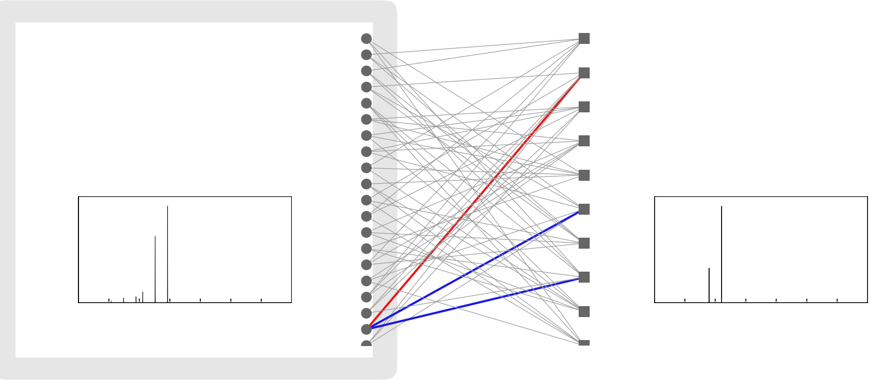

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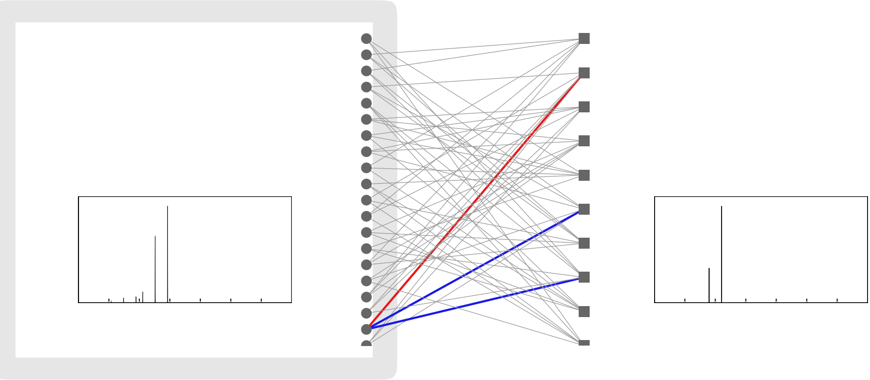

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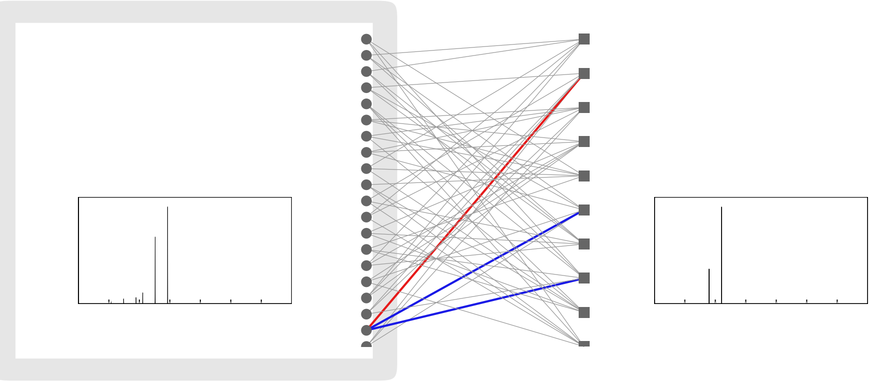

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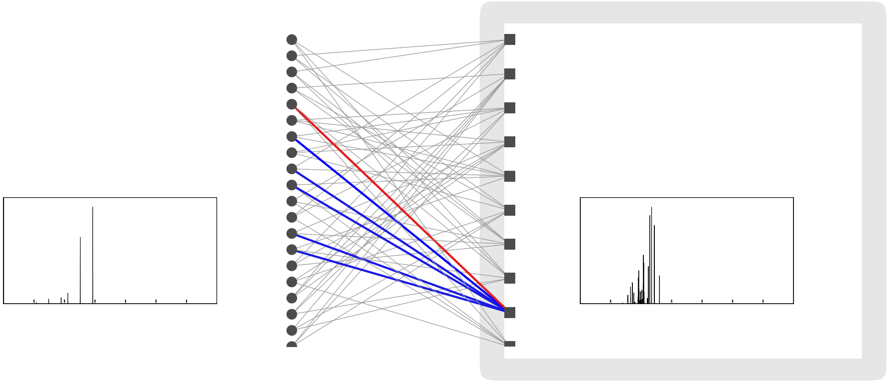

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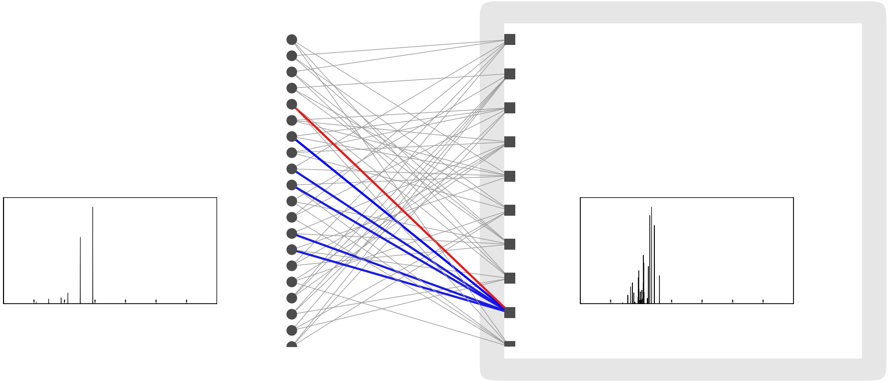

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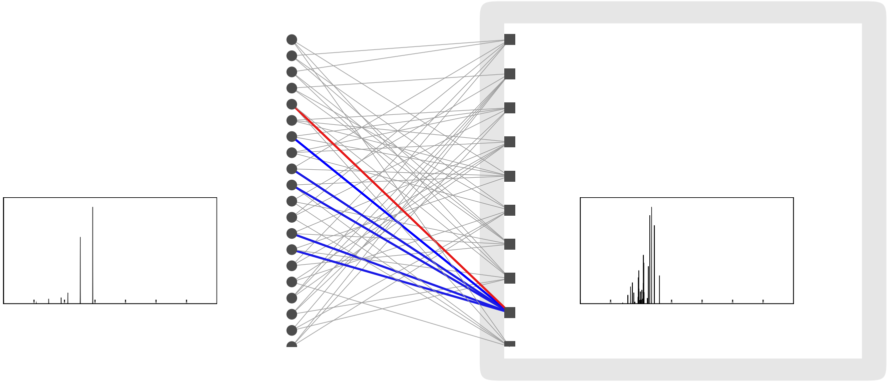

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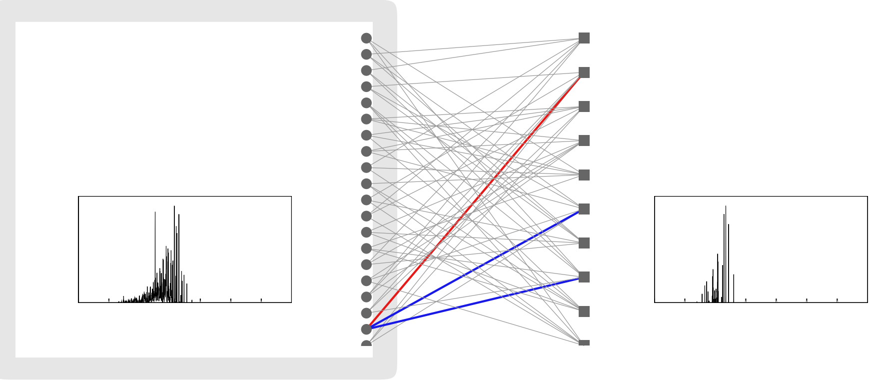

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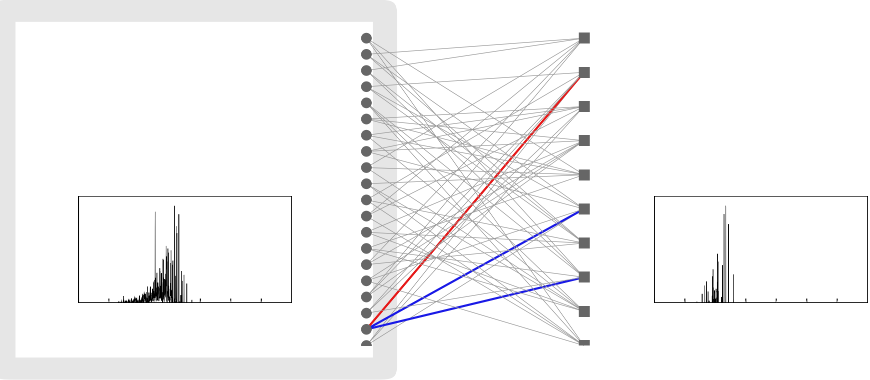

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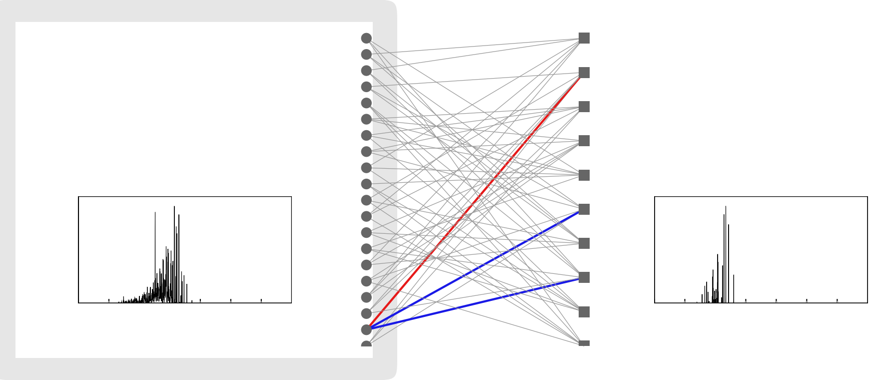

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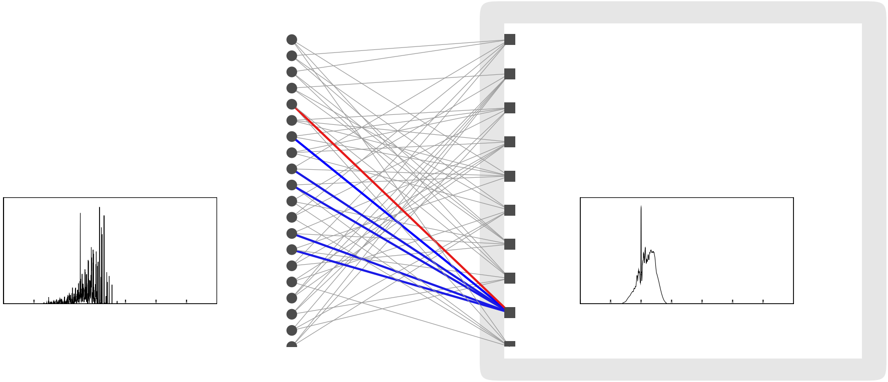

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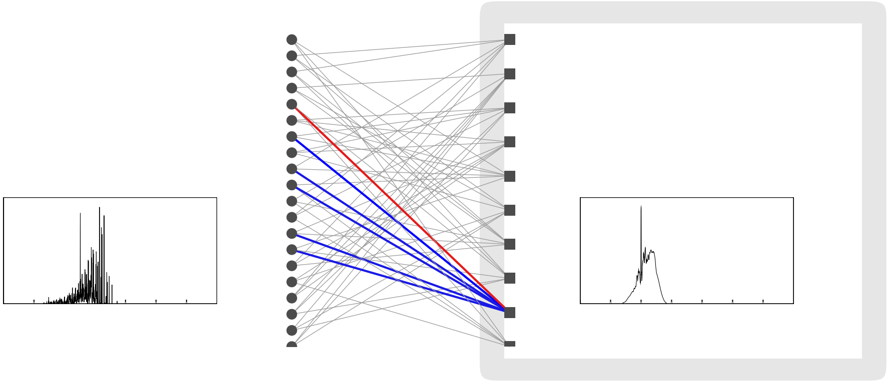

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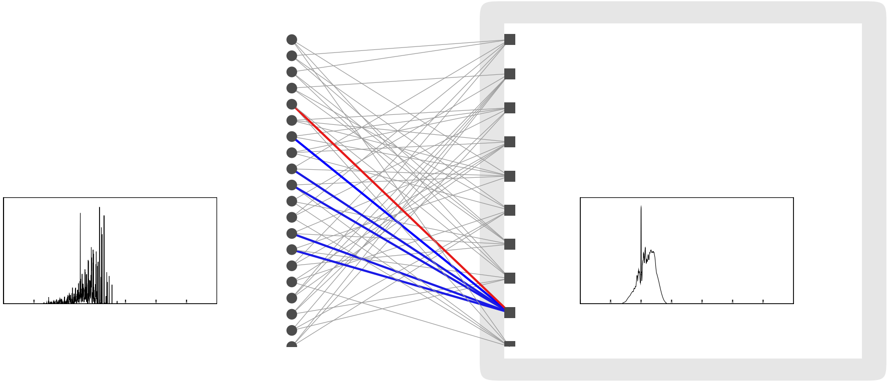

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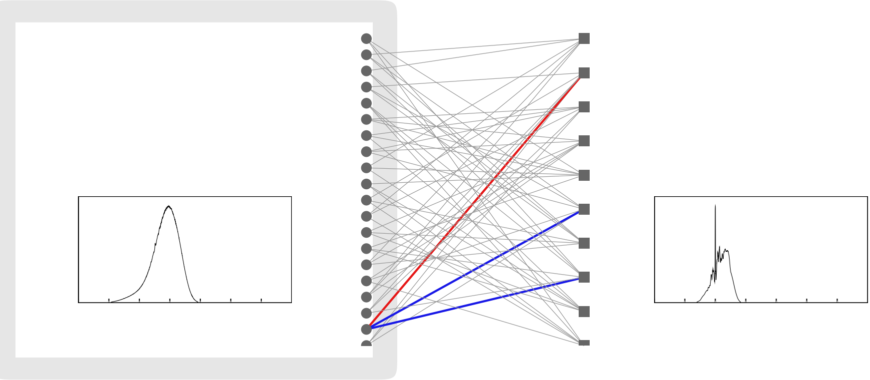

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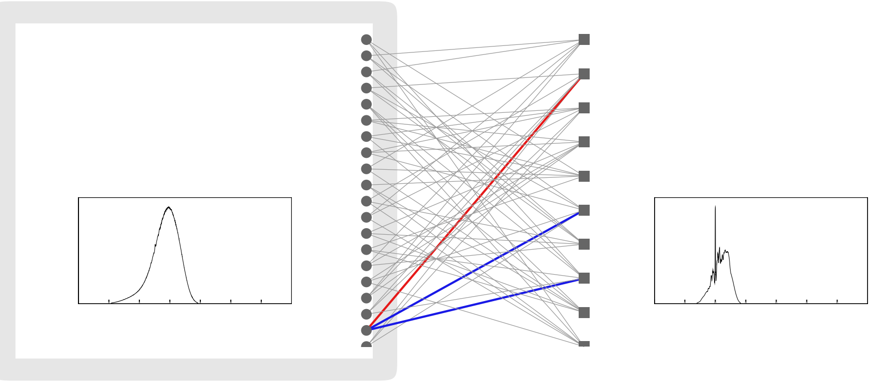

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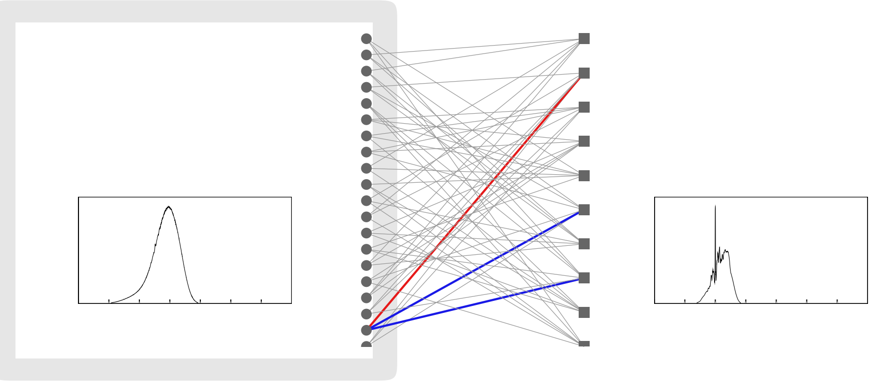

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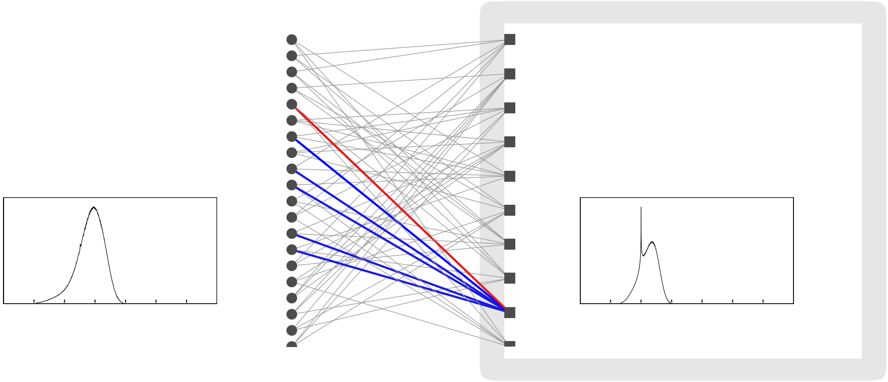

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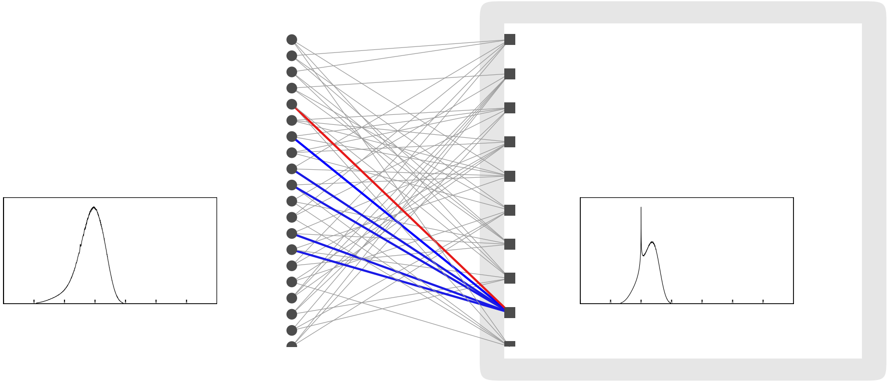

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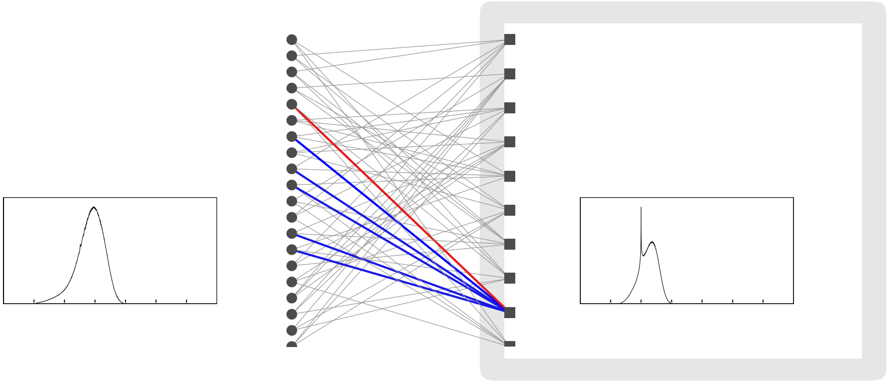

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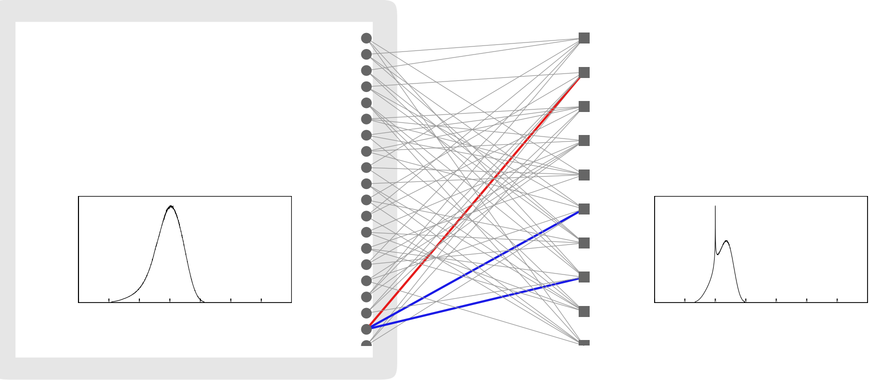

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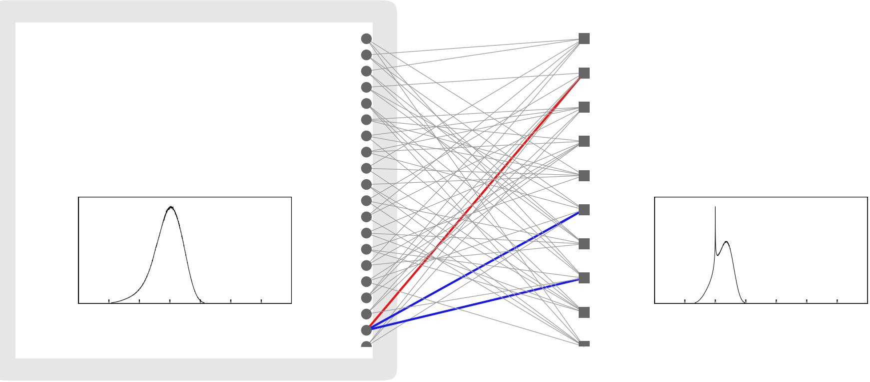

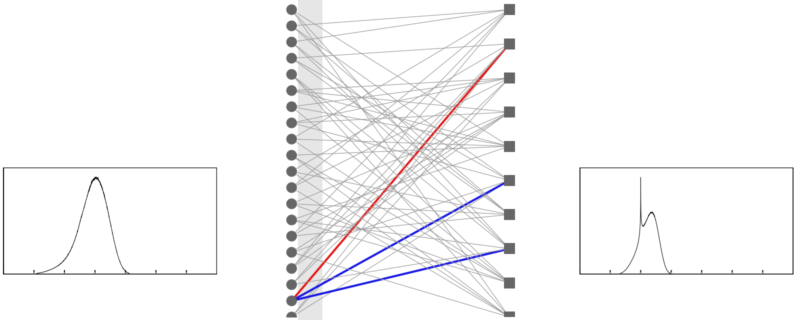

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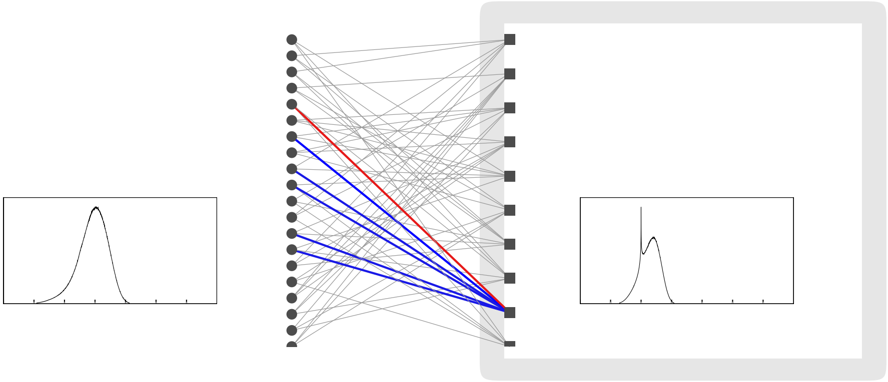

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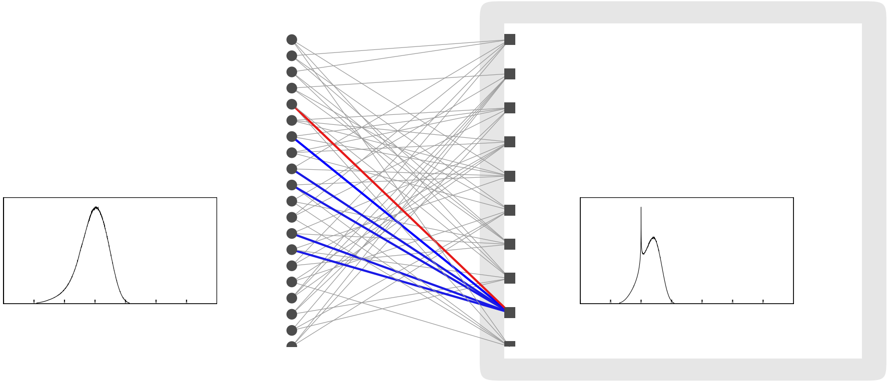

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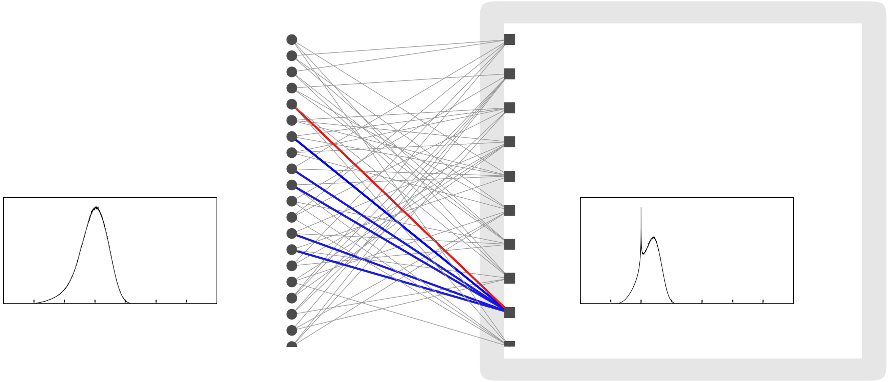

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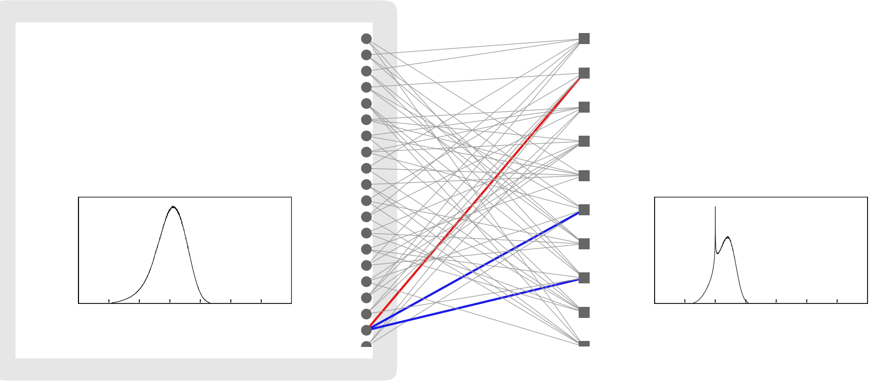

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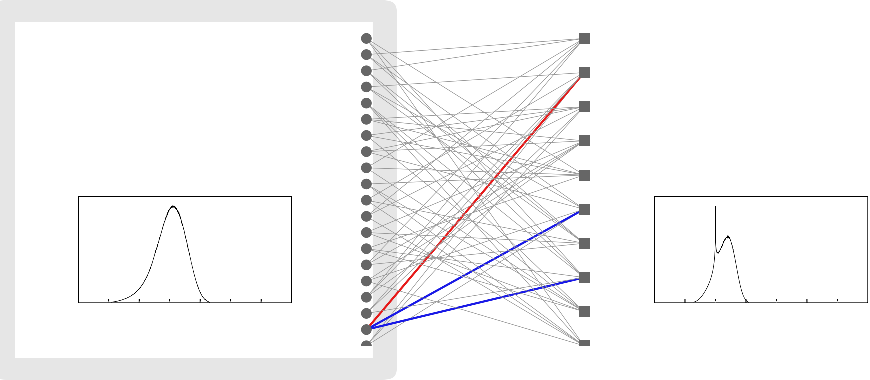

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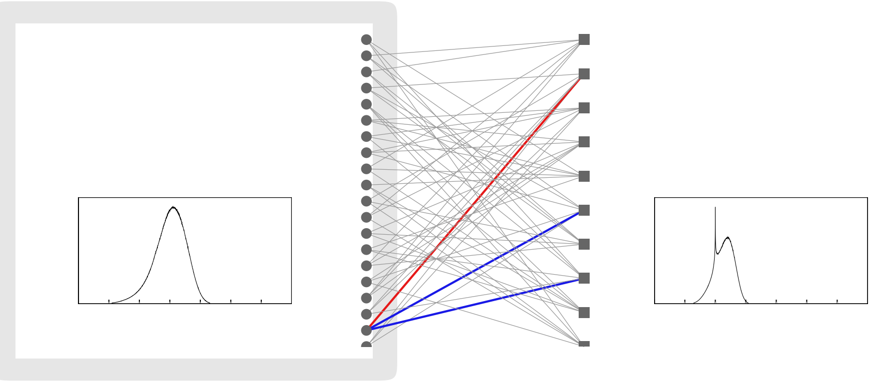

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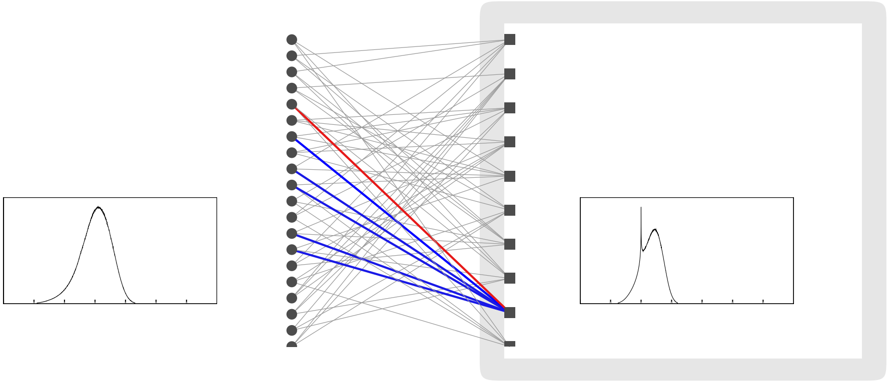

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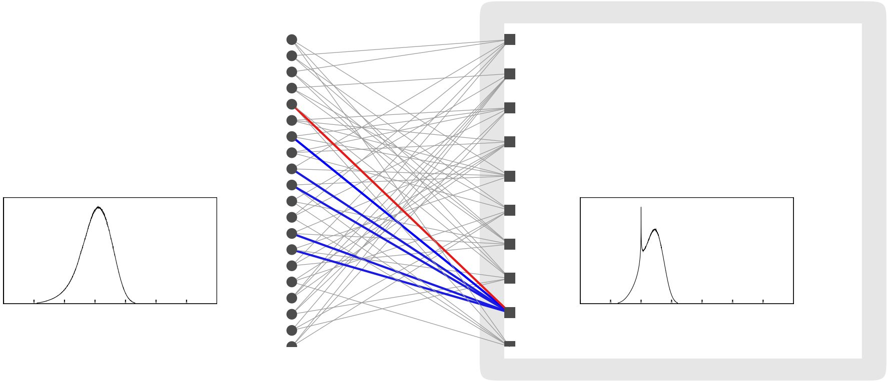

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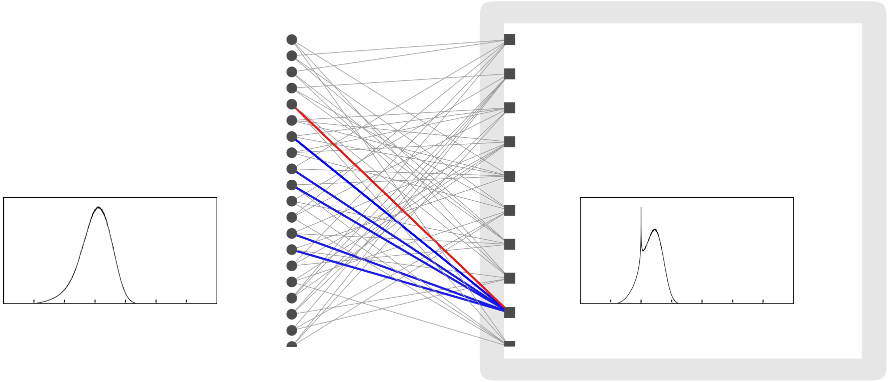

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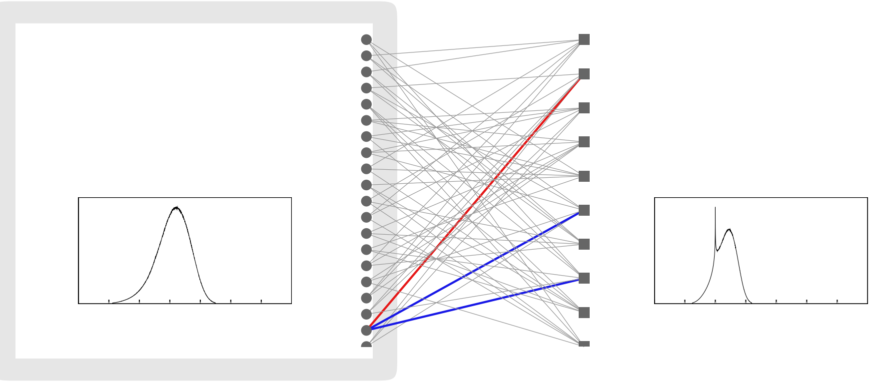

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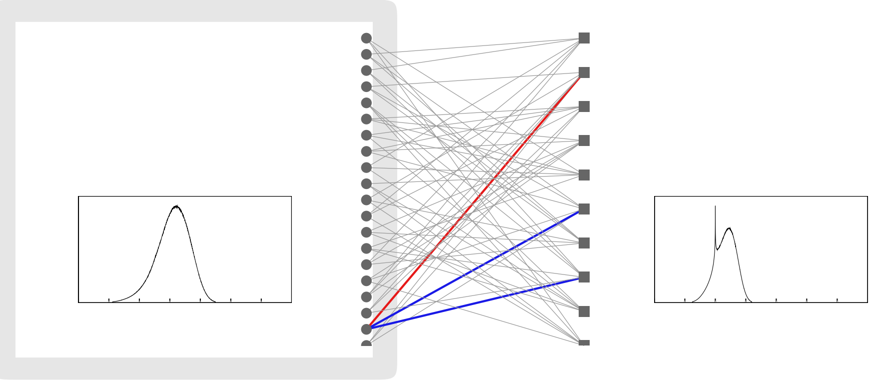

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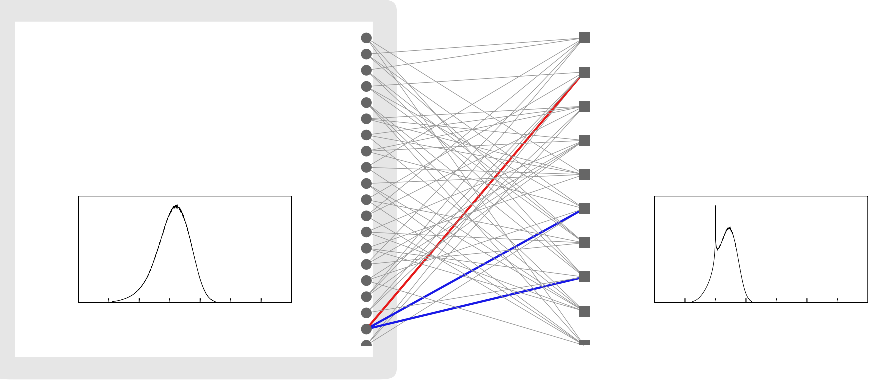

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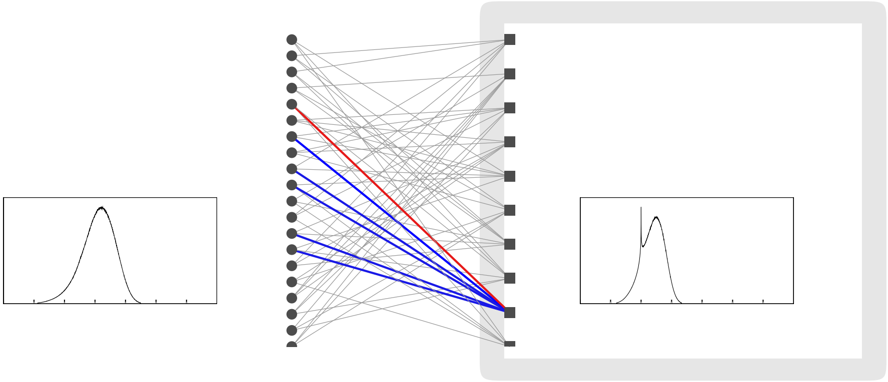
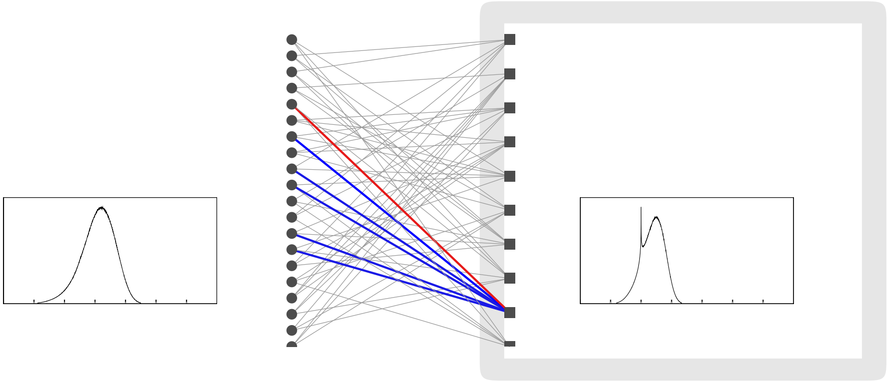


Message Passing + Density evolution analysis



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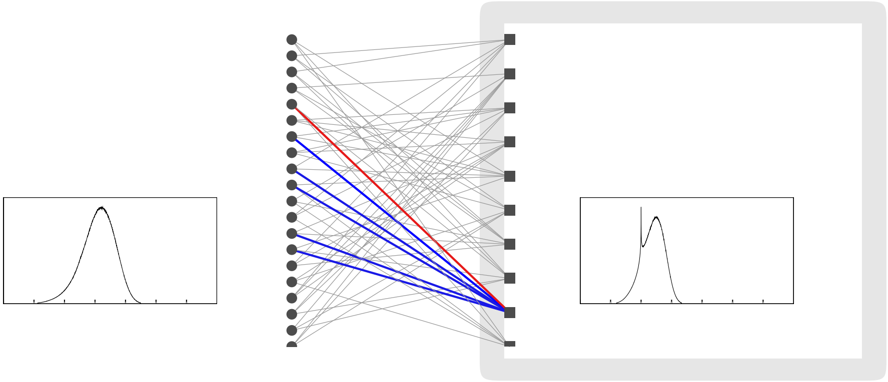

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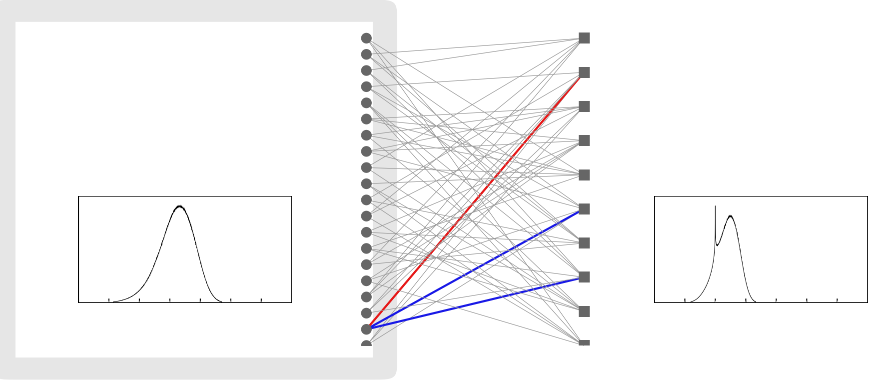

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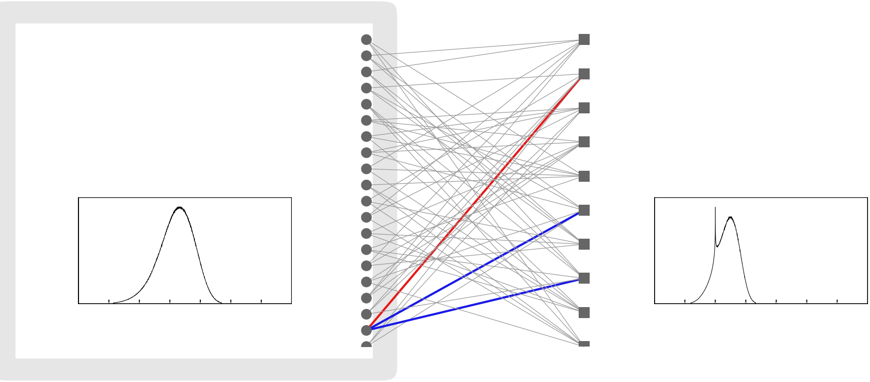

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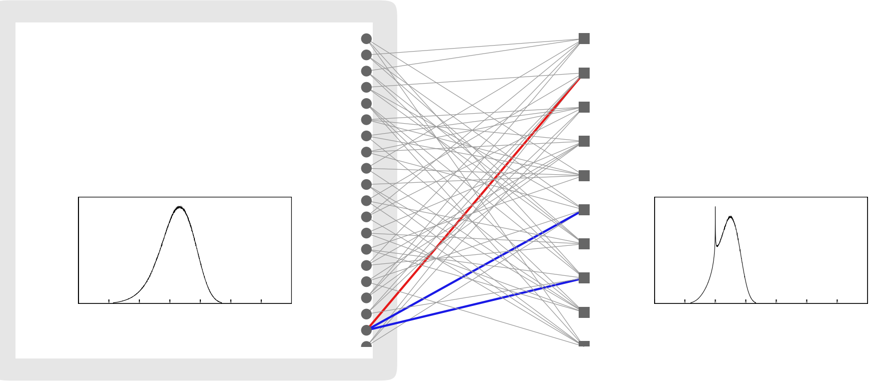

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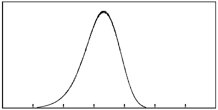

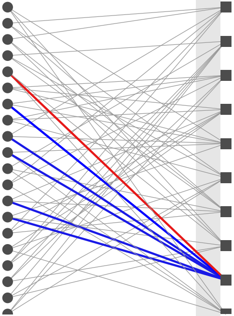

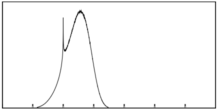

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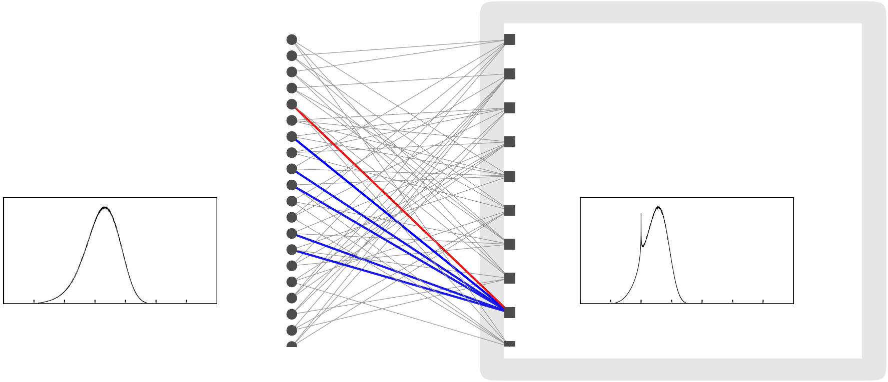

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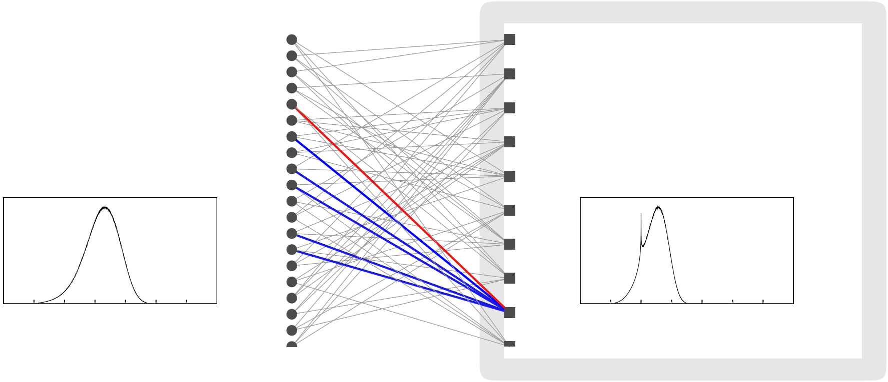

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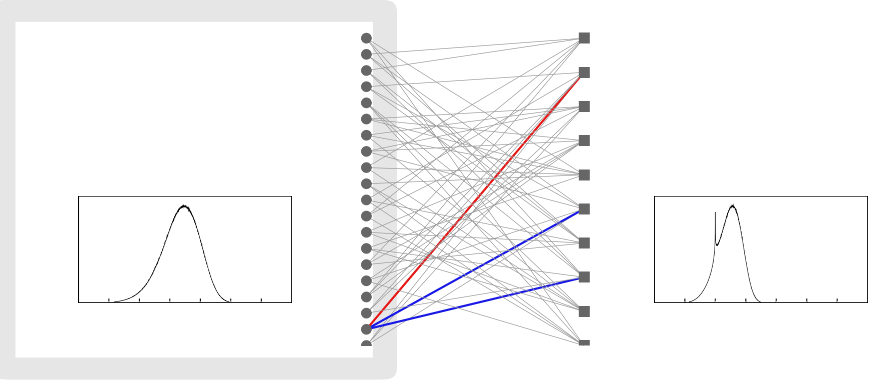

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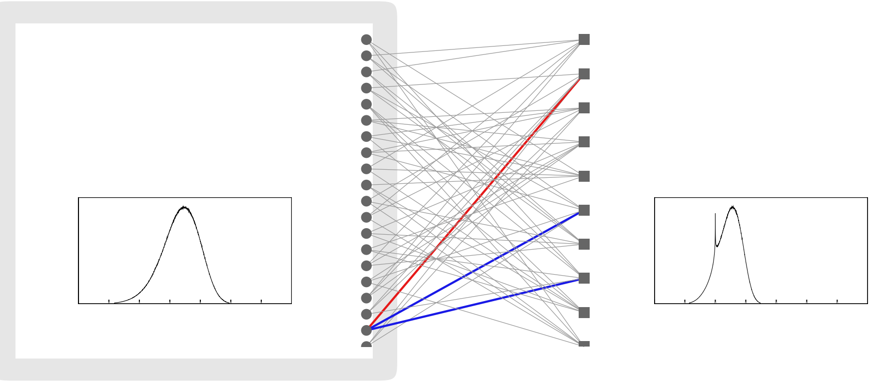

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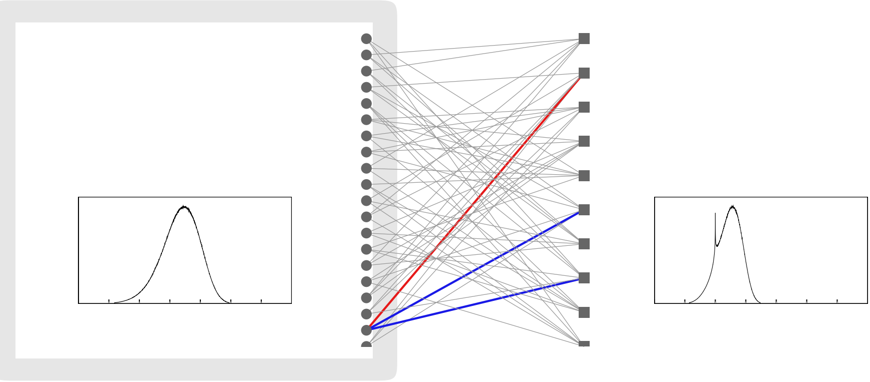

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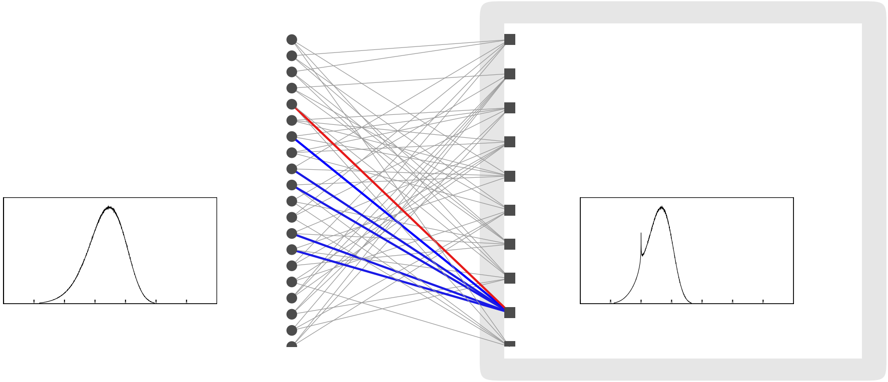

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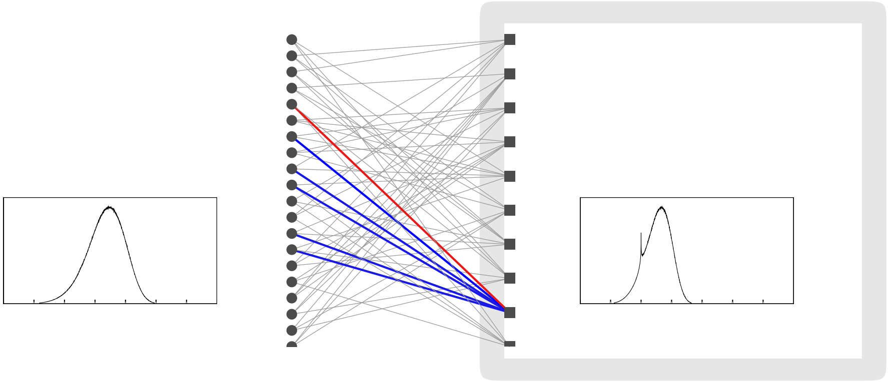

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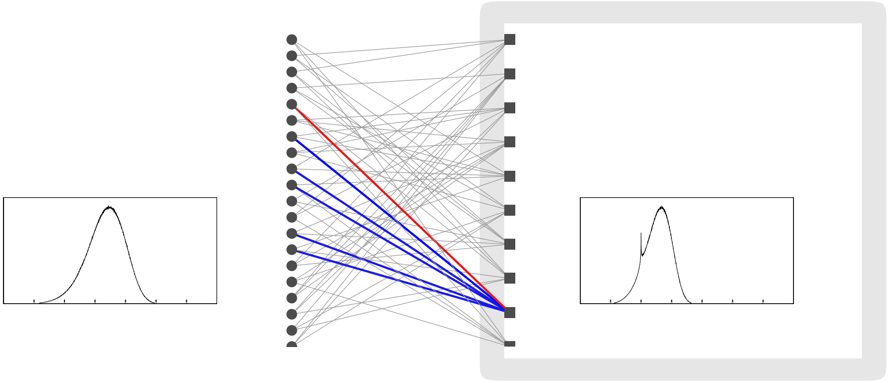

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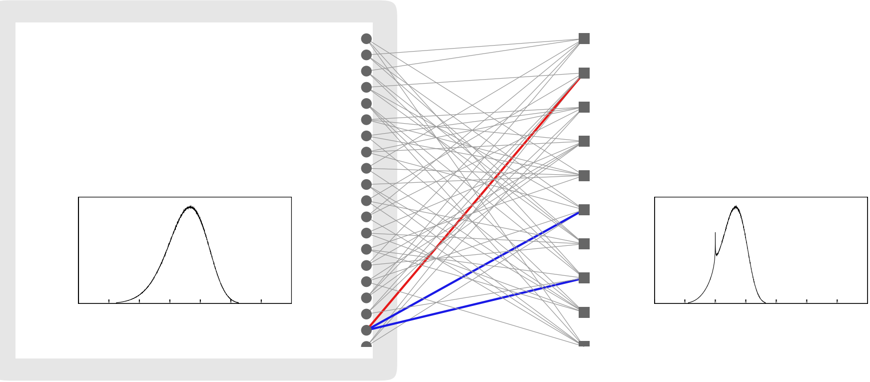

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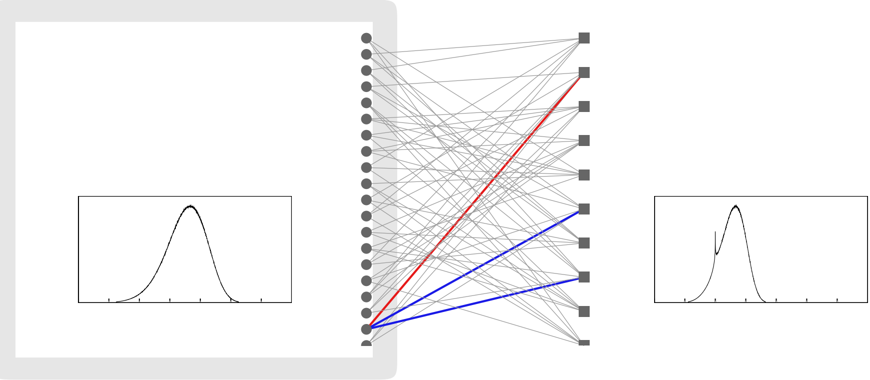

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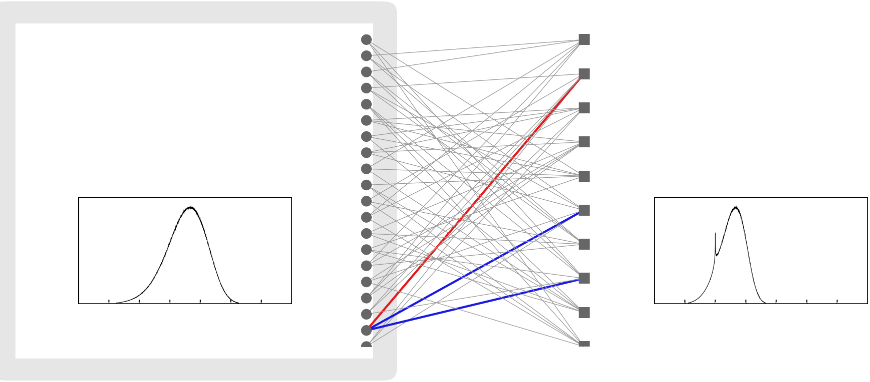

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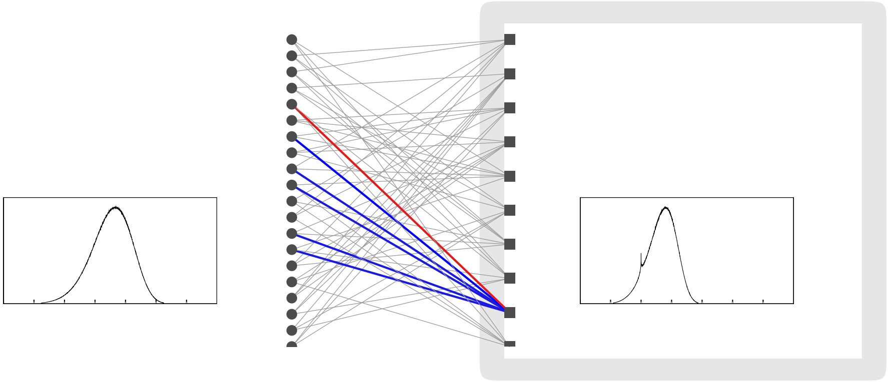

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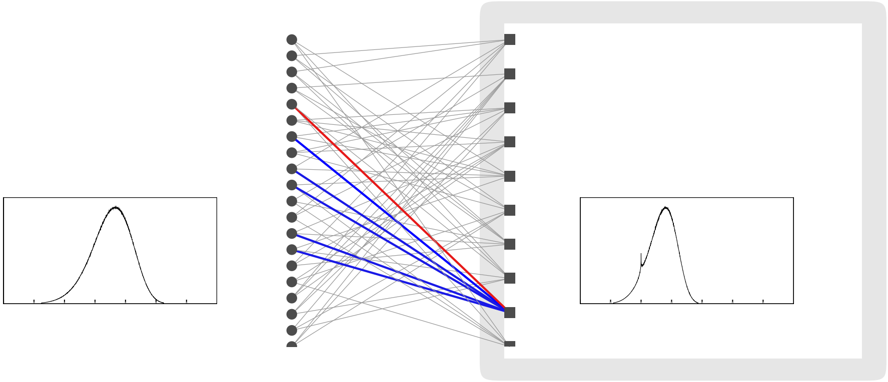

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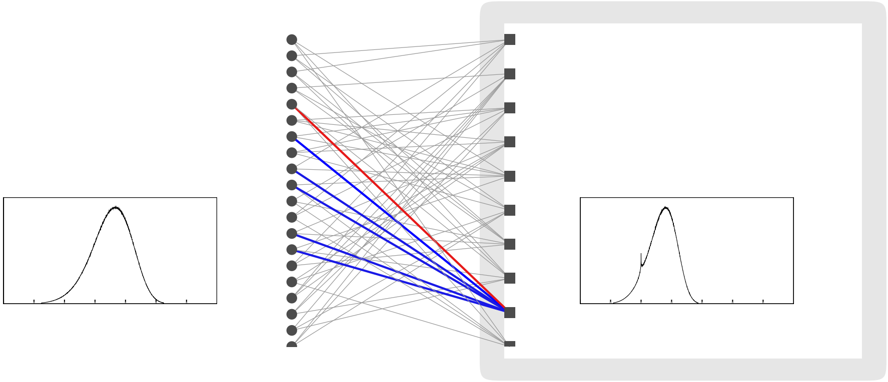

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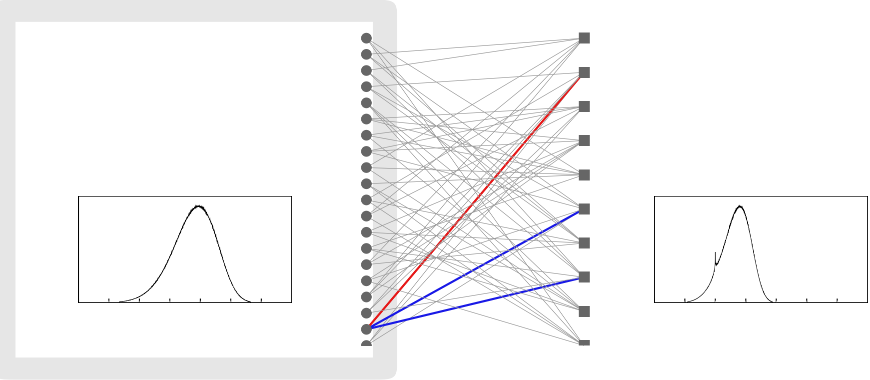

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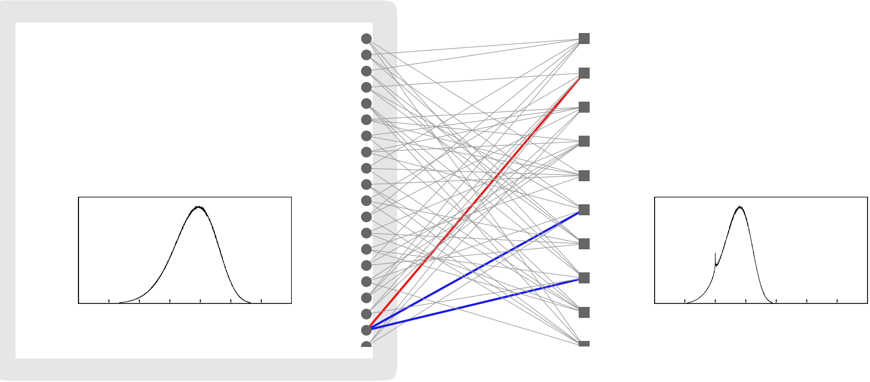

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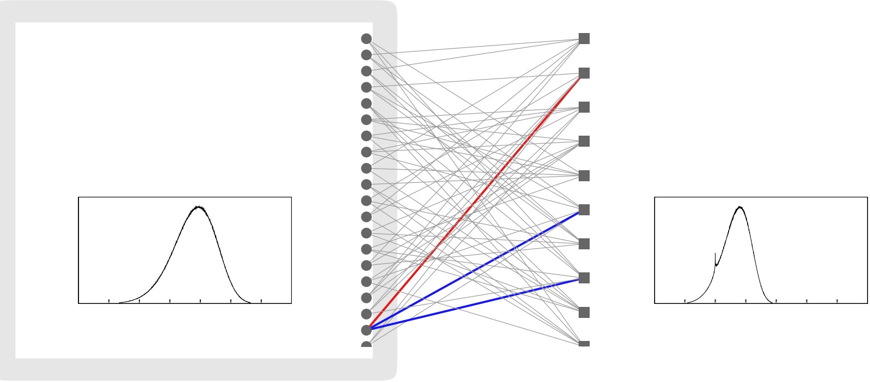

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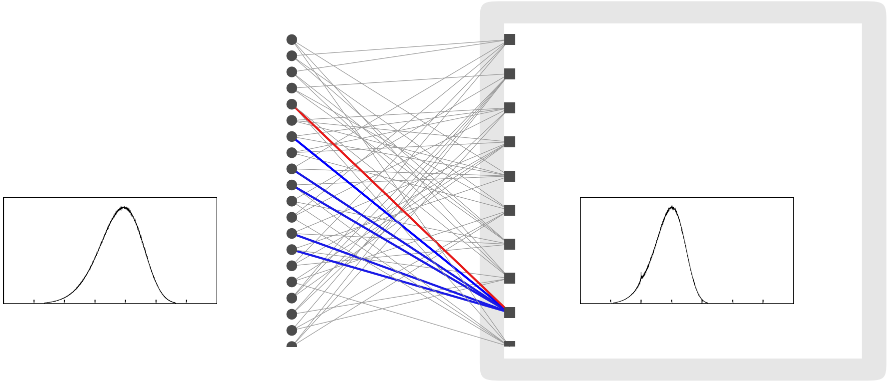

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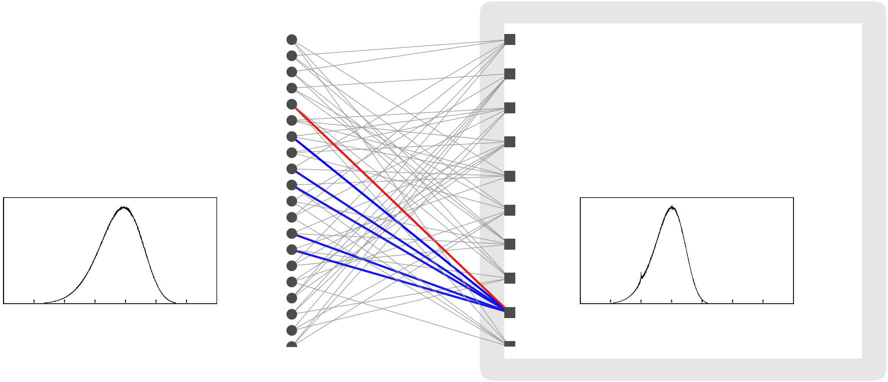

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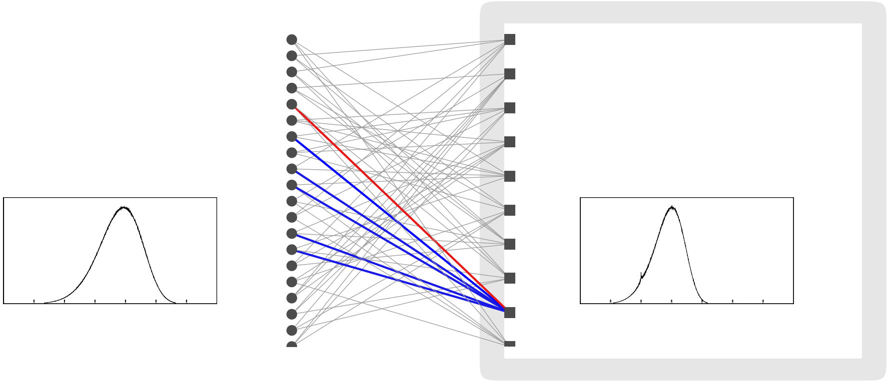

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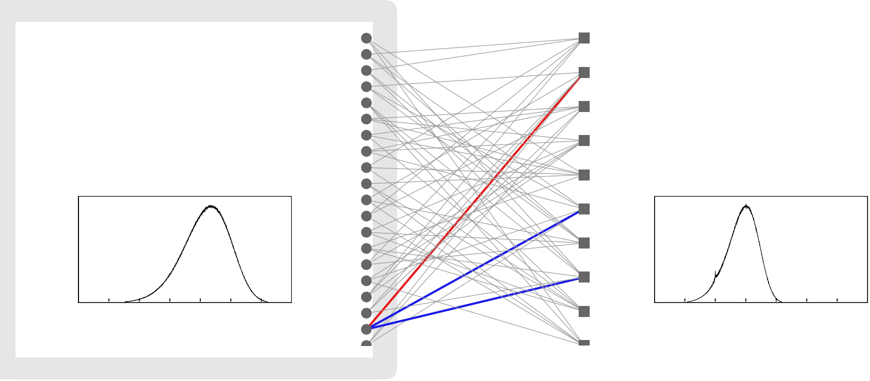

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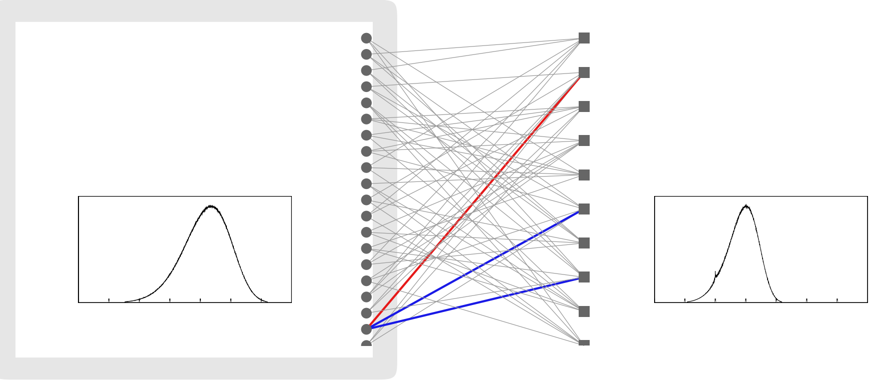

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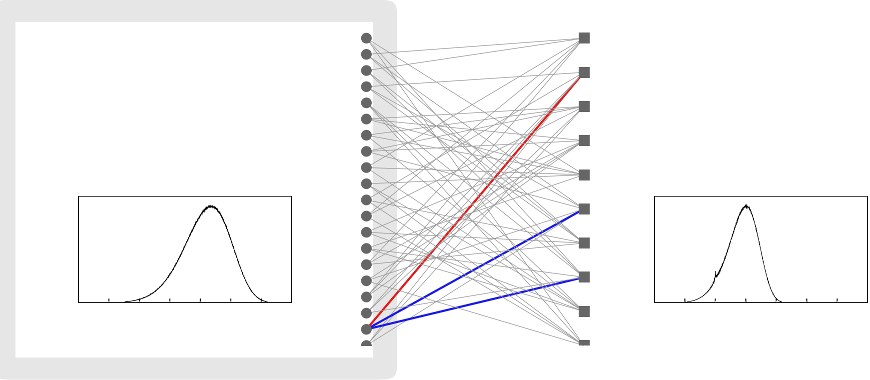

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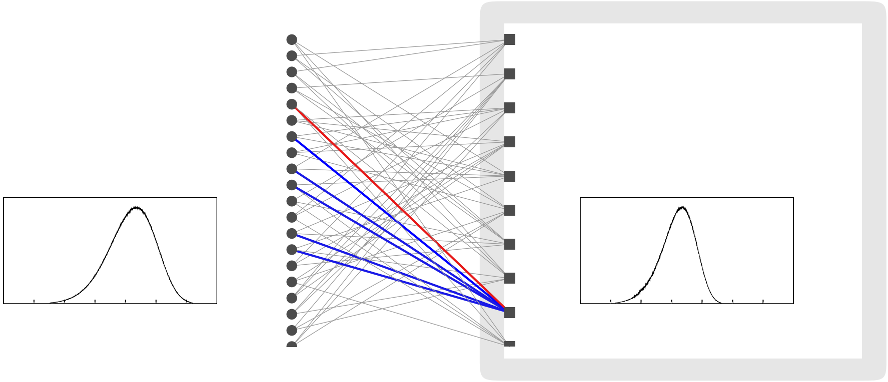

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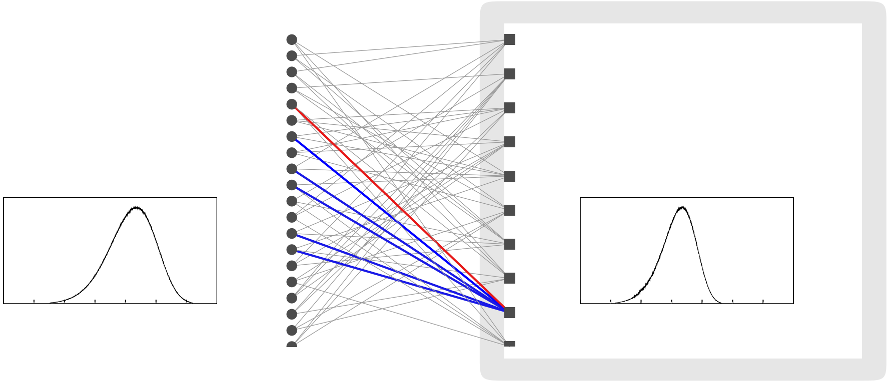

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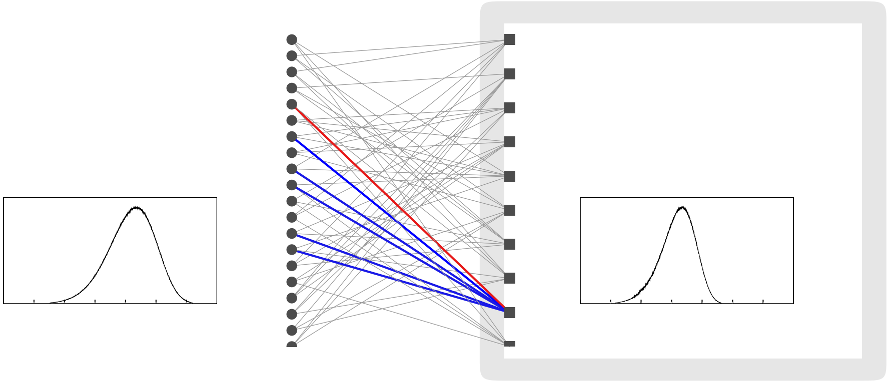

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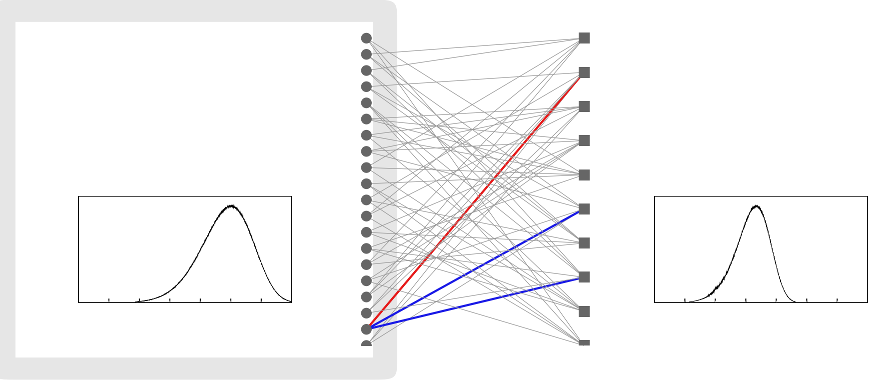

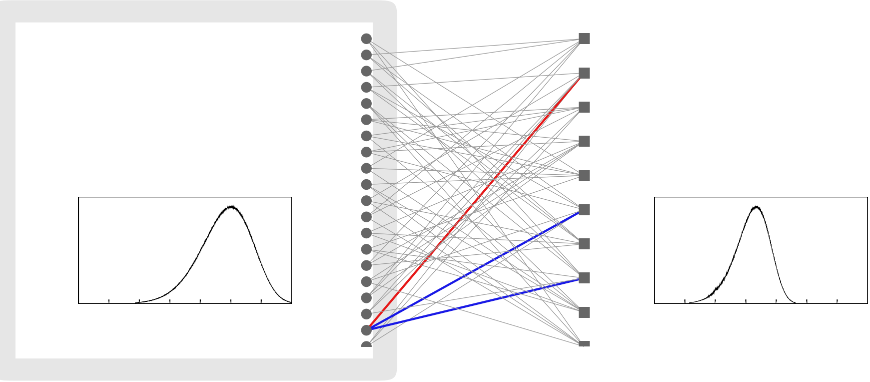

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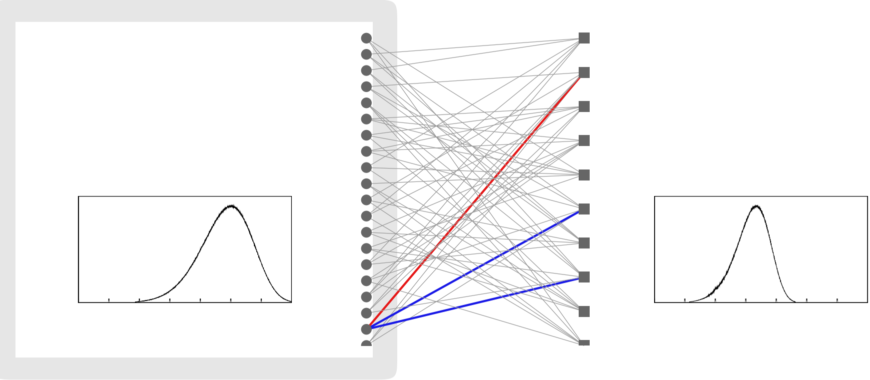

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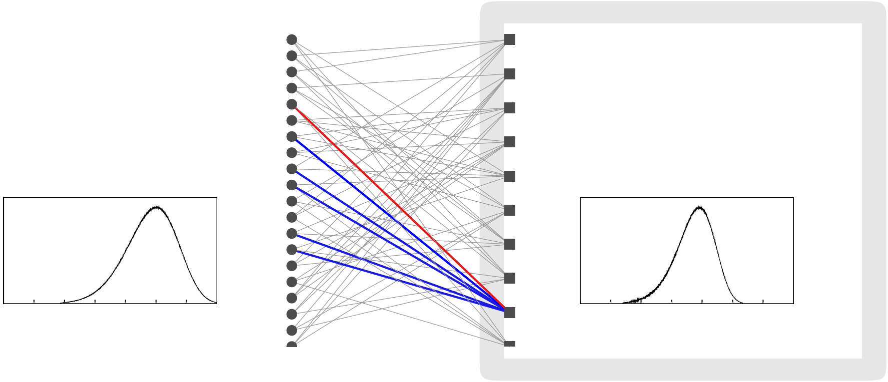
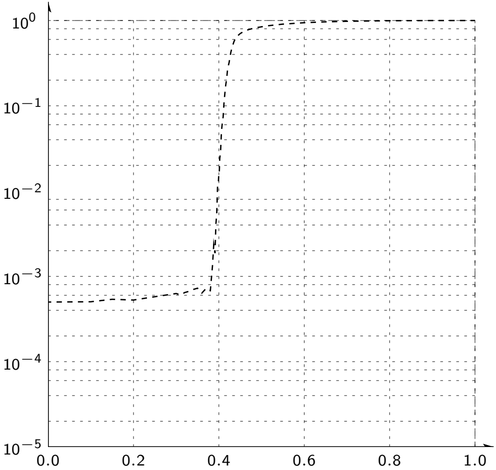


decoding error probability

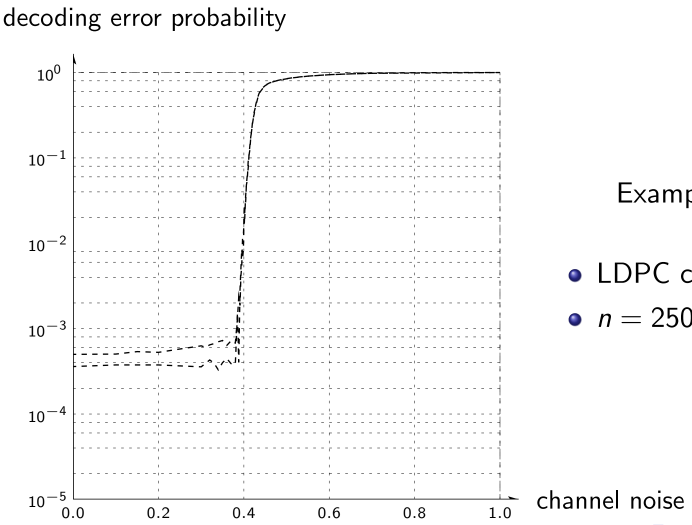


channel noise

Example:

- LDPC codes
- $n = 2500$

Average (Message Passing) Performance



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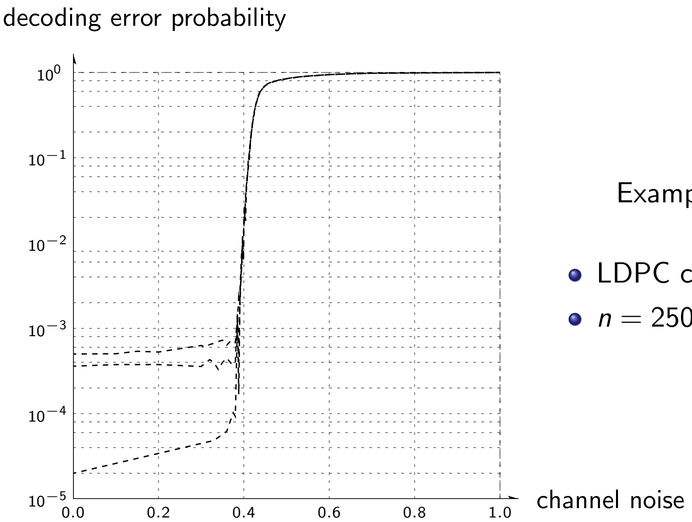

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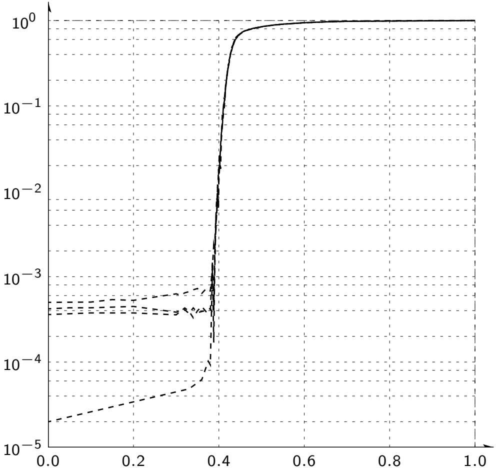

channel noise

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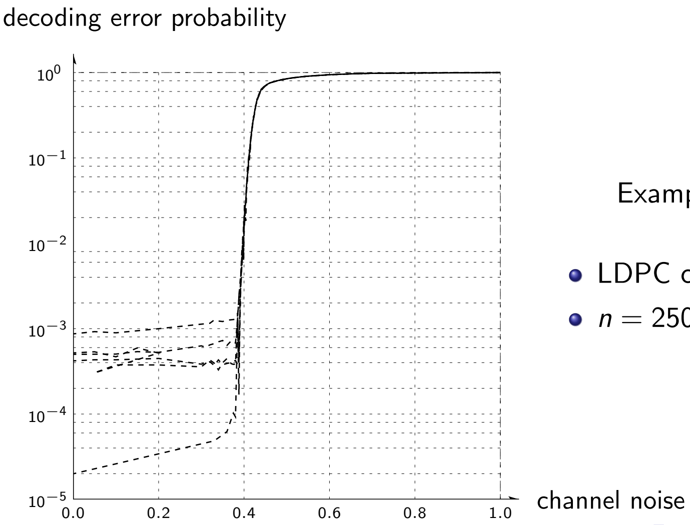

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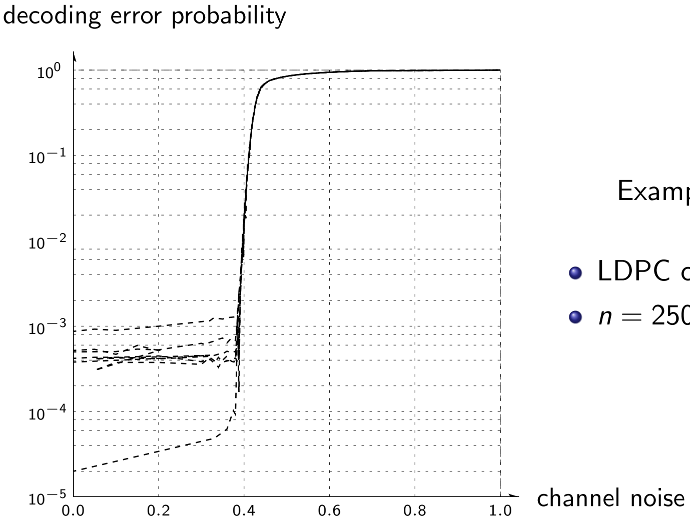

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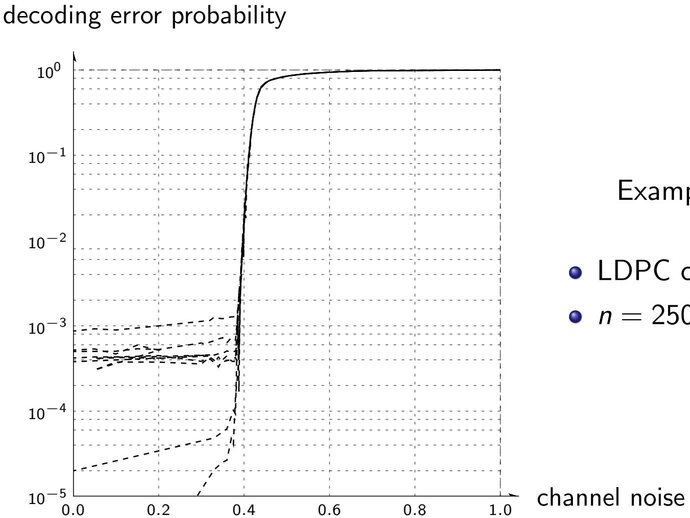

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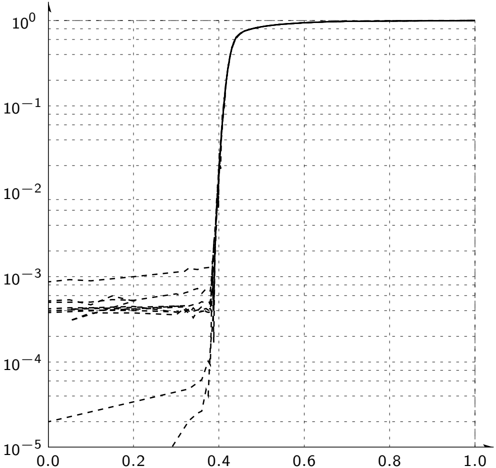

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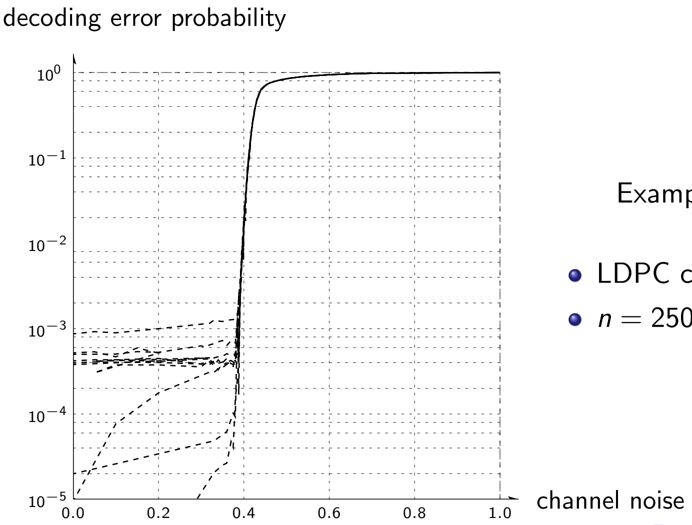

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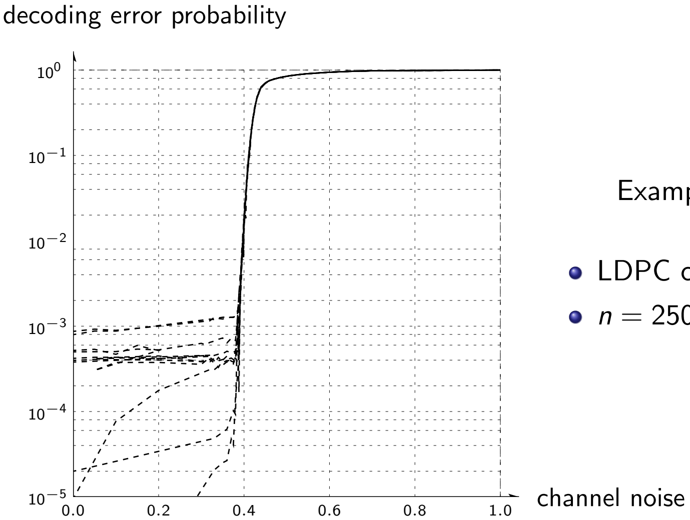

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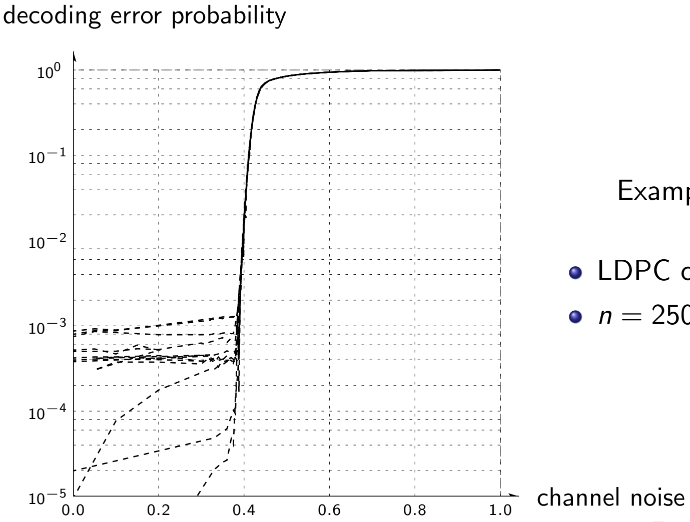

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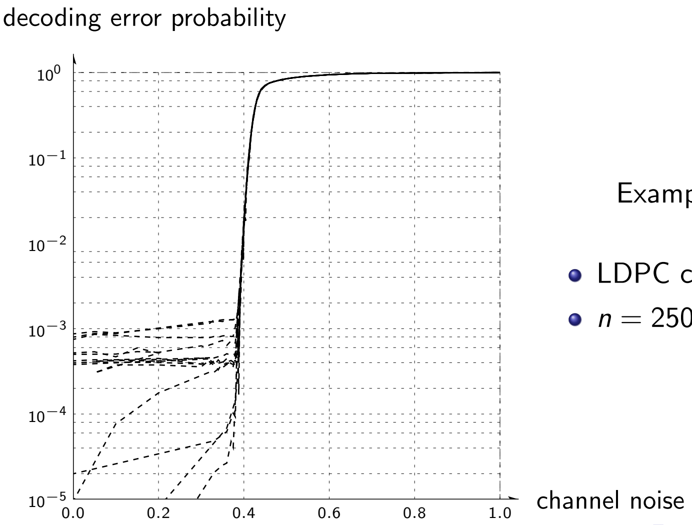

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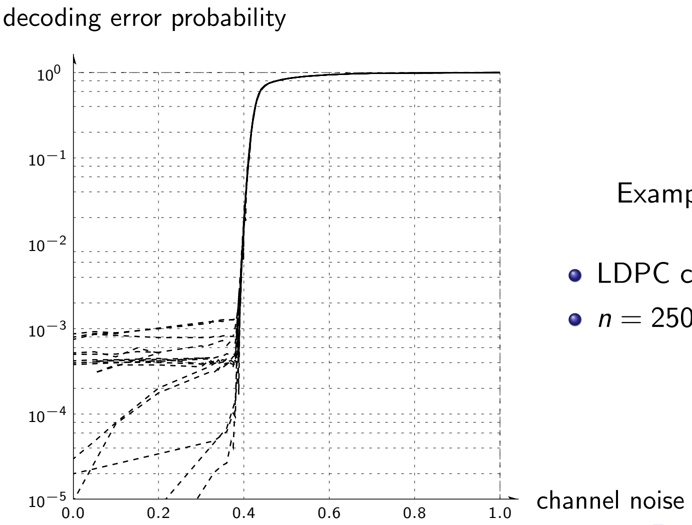

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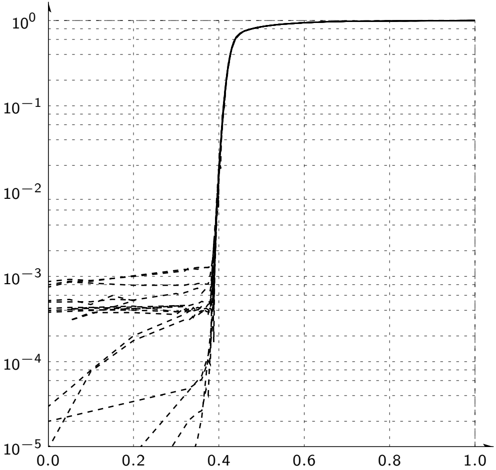

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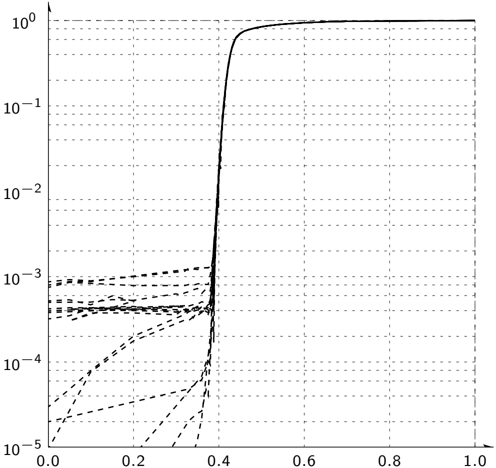

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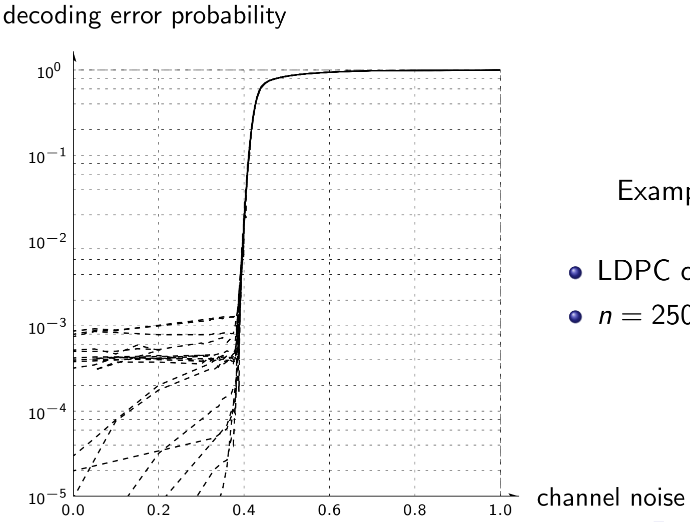

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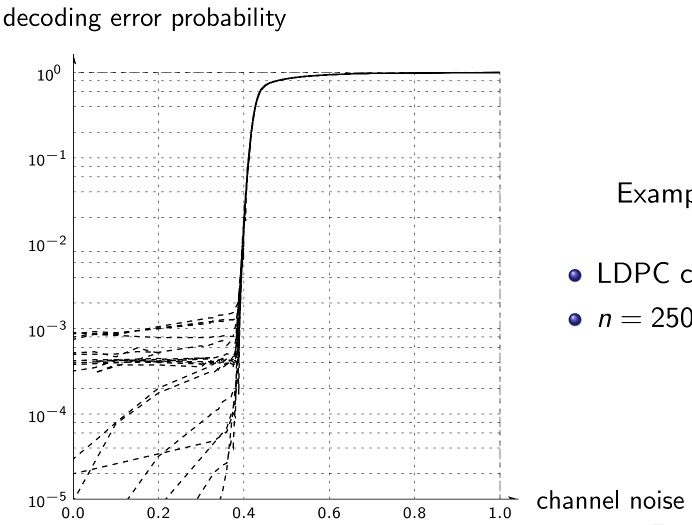

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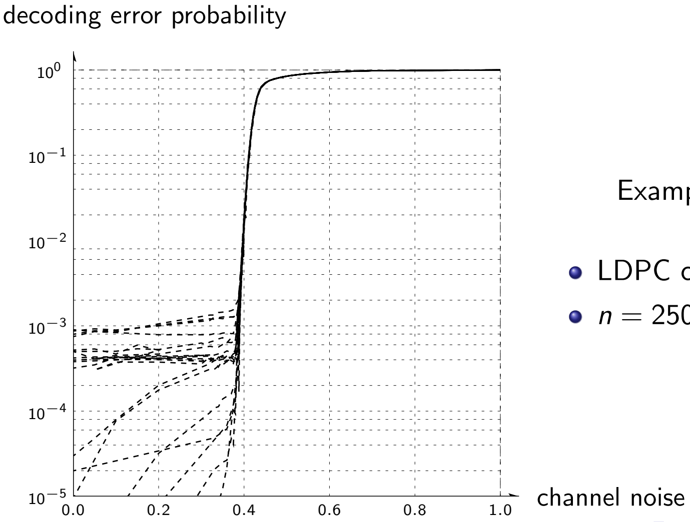

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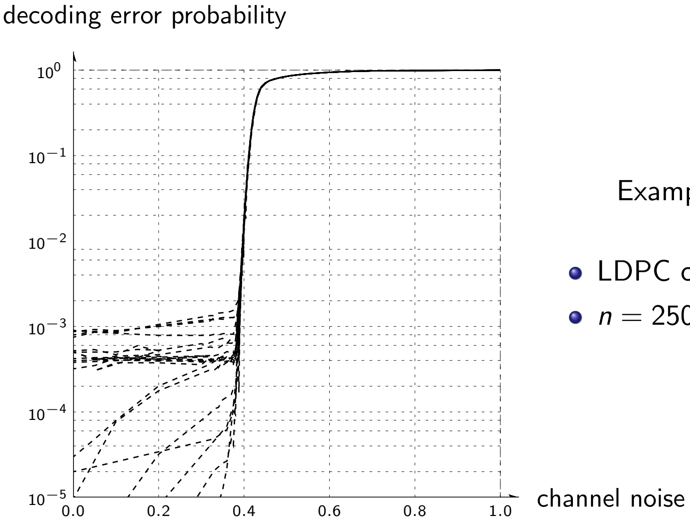

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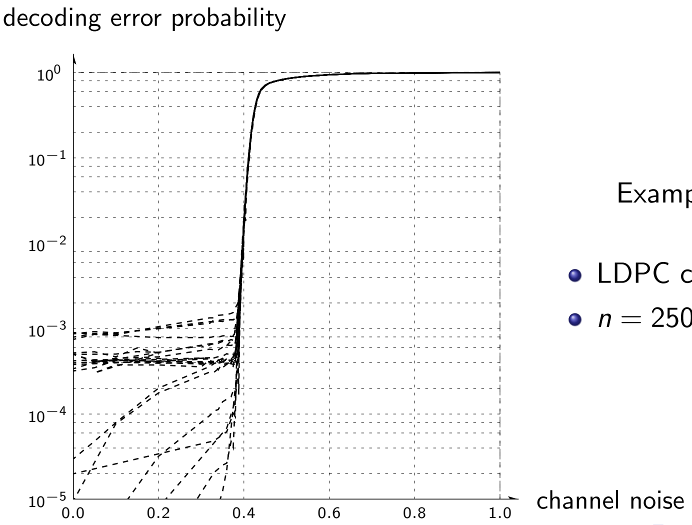

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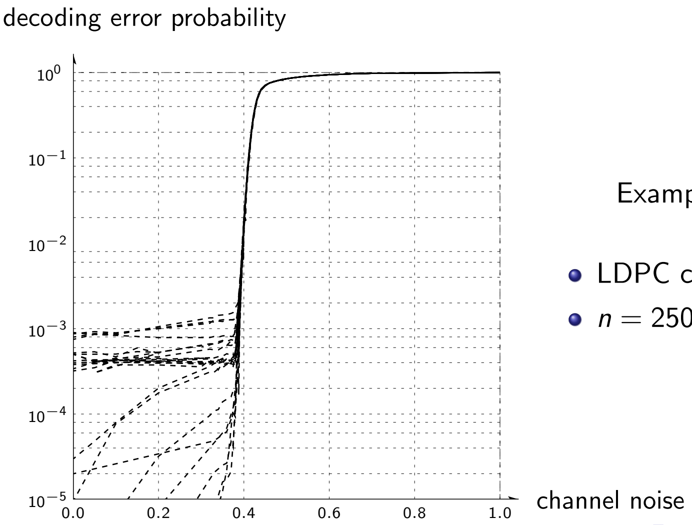

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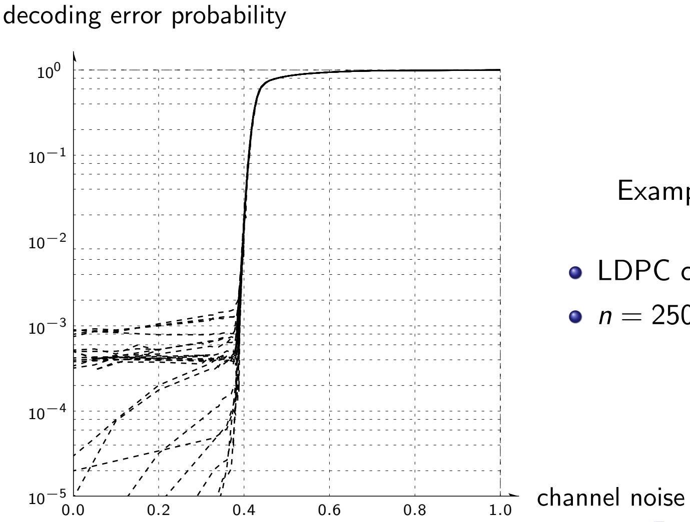

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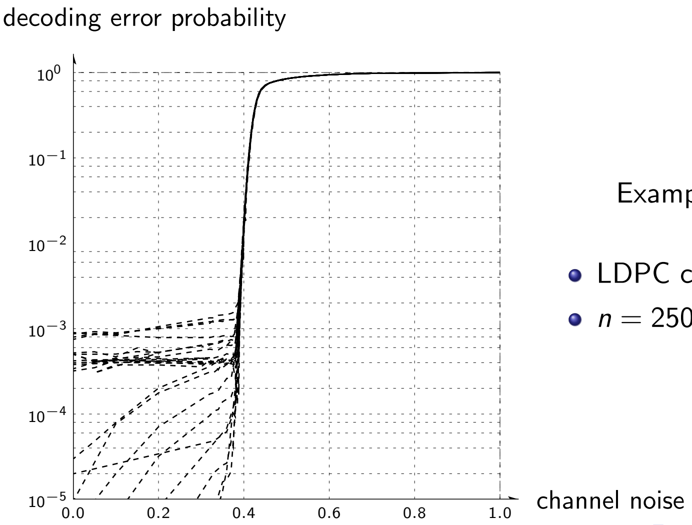

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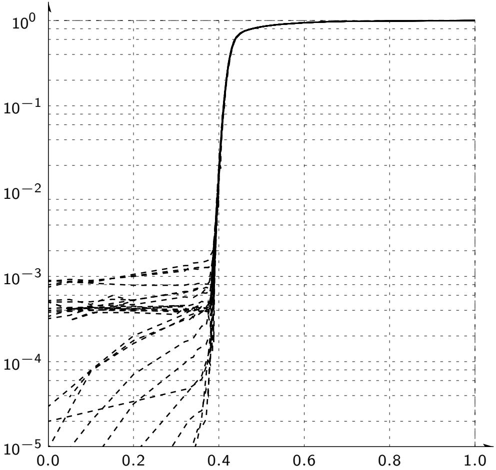

channel noise

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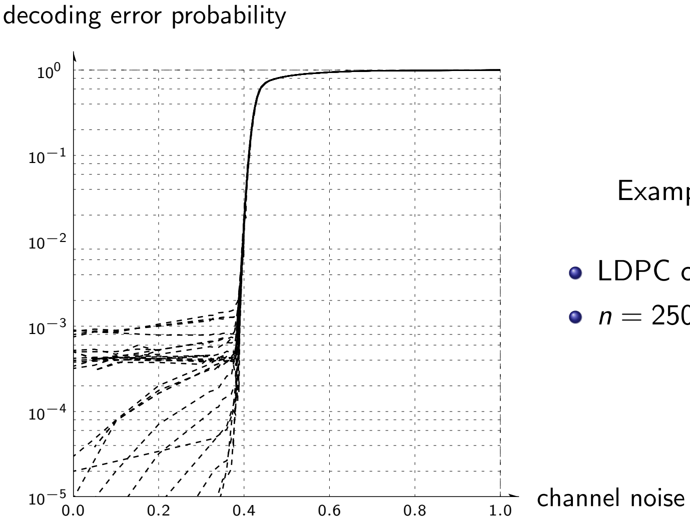

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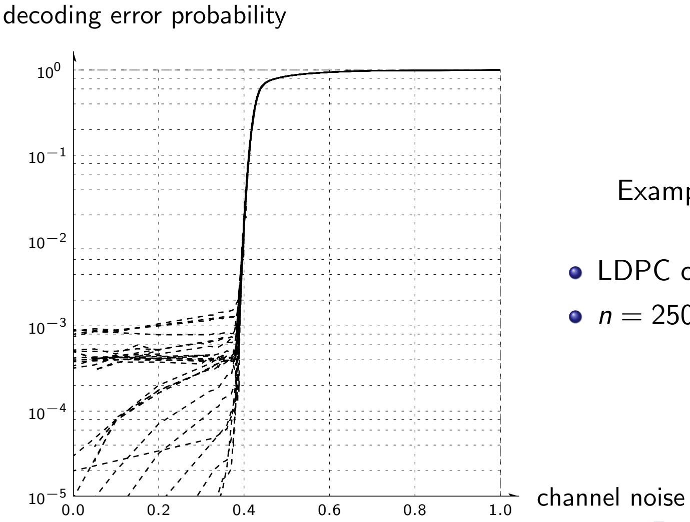

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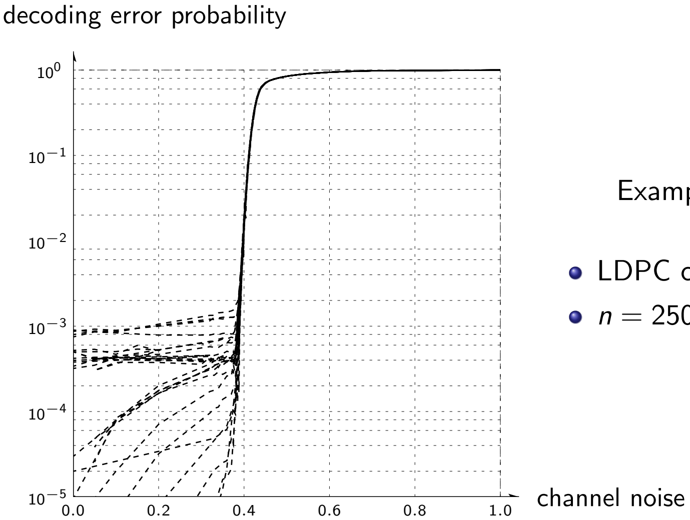

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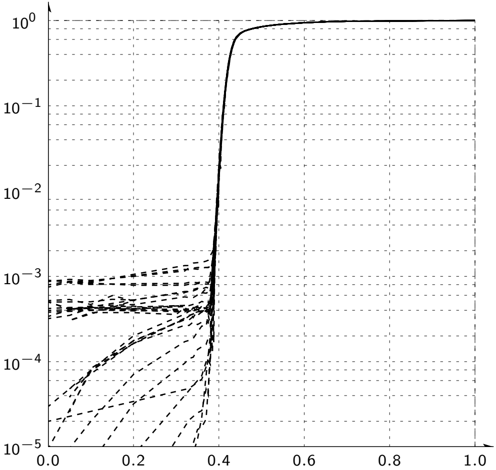

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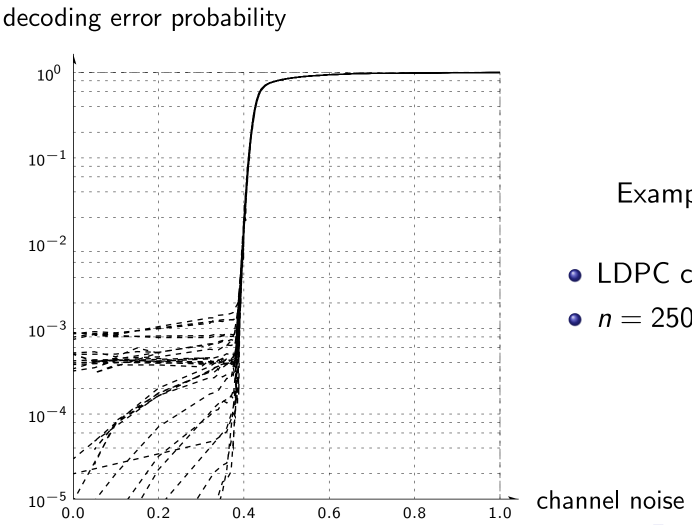
- $n = 2500$

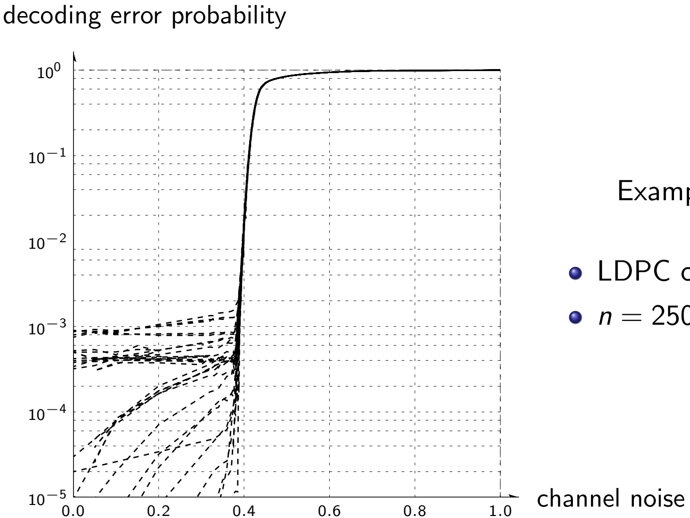
Average (Message Passing) Performance



Example:

- LDPC codes
- $n = 2500$

Average (Message Passing) Performance



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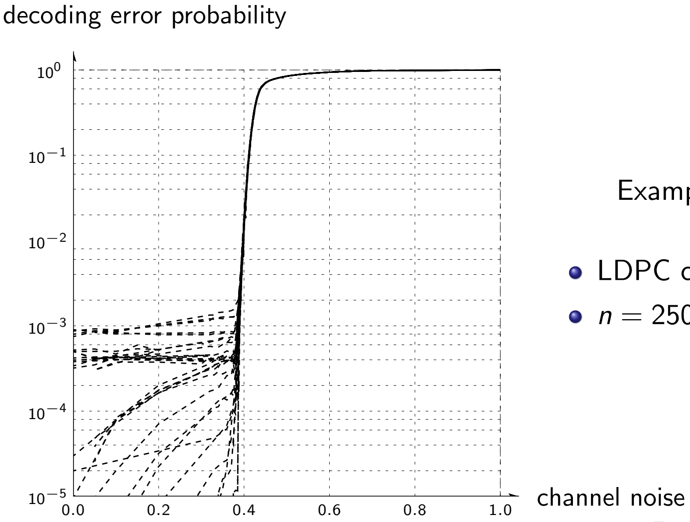
- $n = 2500$

Average (Message Passing) Performance



Example:

- LDPC codes
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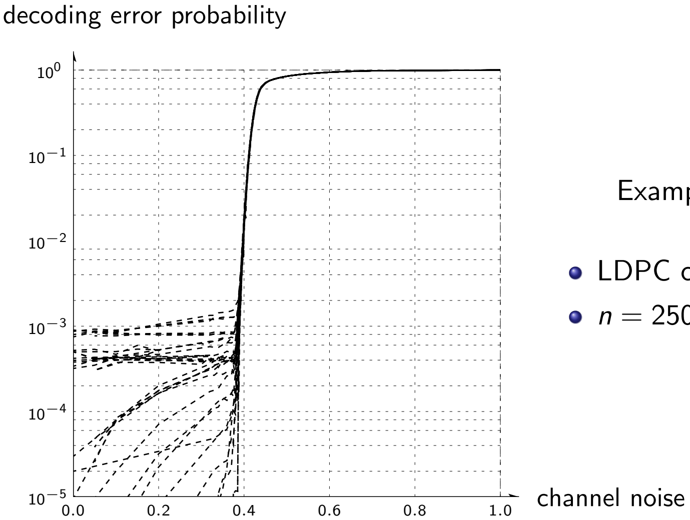
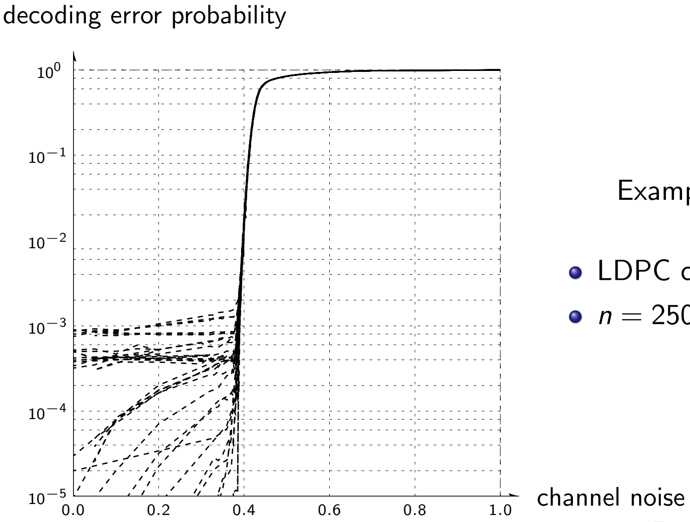


Example:

- LDPC codes
- $n = 2500$



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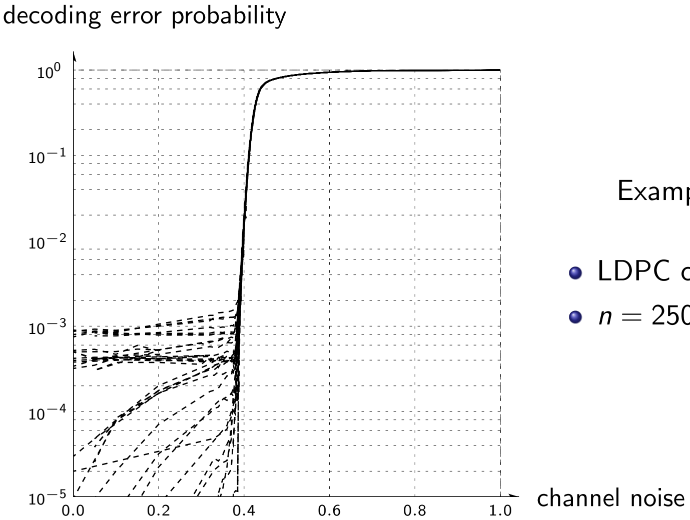

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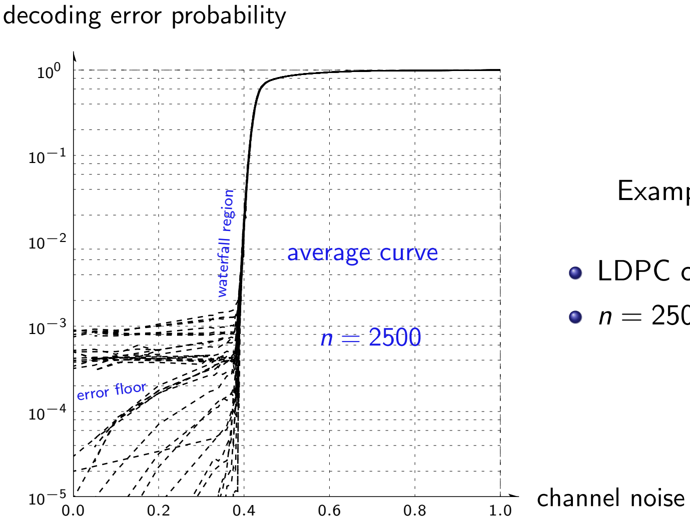

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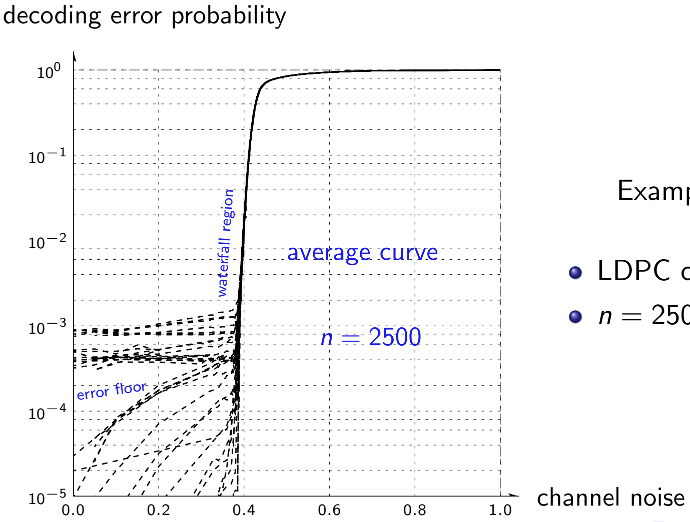

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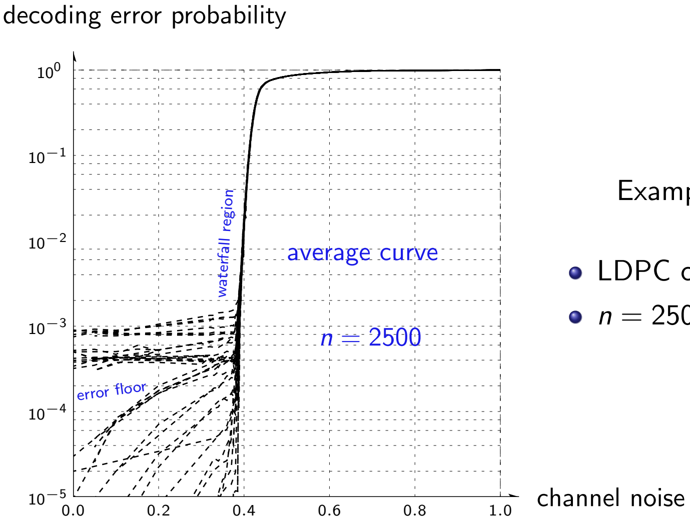
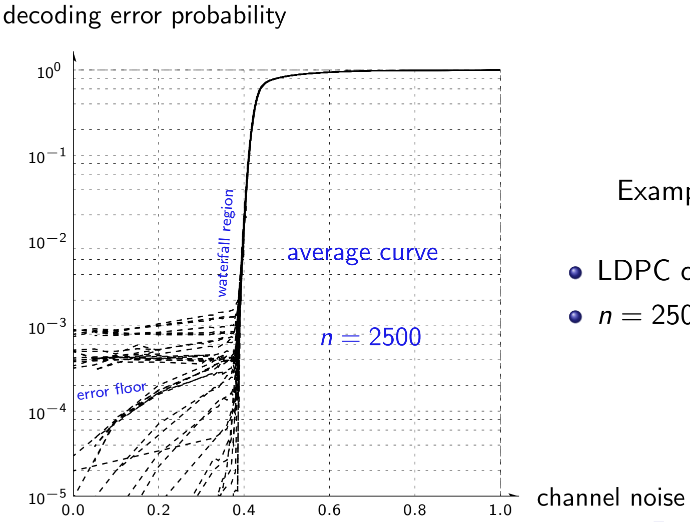
- $n = 2500$

Average (Message Passing) Performance



Example:

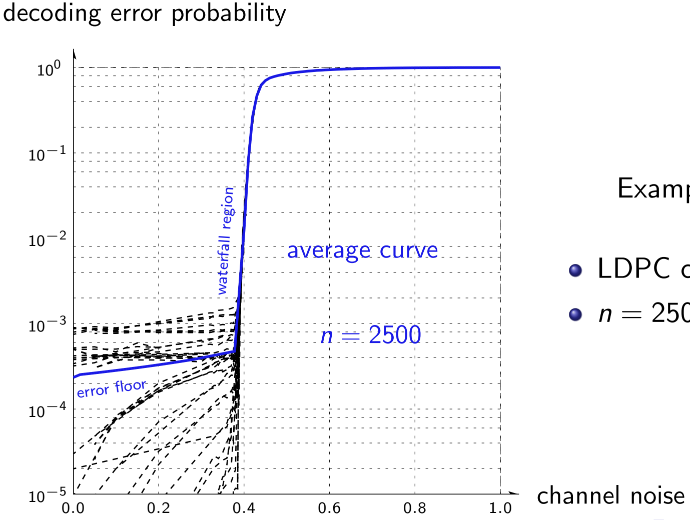
- LDPC codes
- $n = 2500$



Example:

- LDPC codes
- $n = 2500$

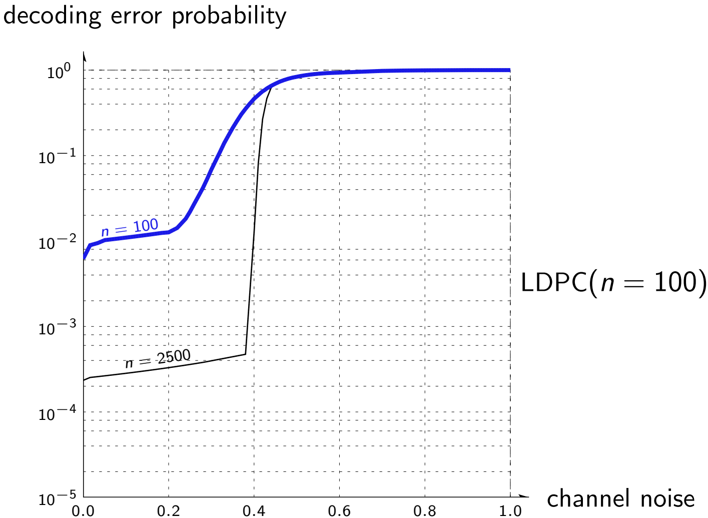
Average (Message Passing) Performance



Example:

- LDPC codes
- $n = 2500$

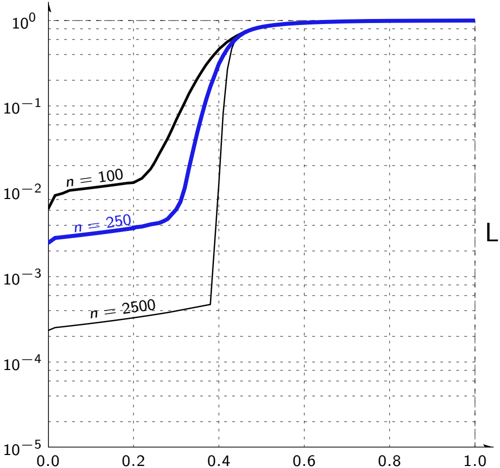
Asymptotic Average (Message Passing) Performance



LDPC($n = 100$)

Asymptotic Average (Message Passing) Performance

decoding error probability

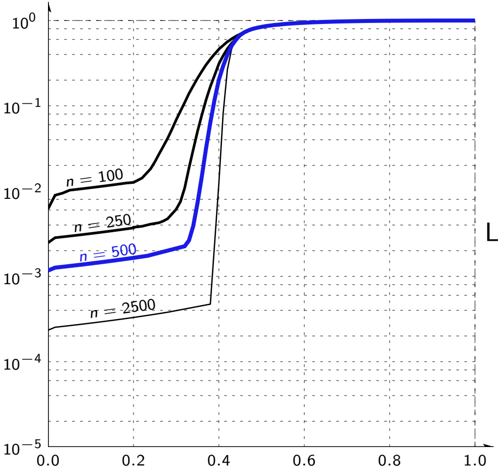


LDPC($n = 250$)

channel noise

Asymptotic Average (Message Passing) Performance

decoding error probability

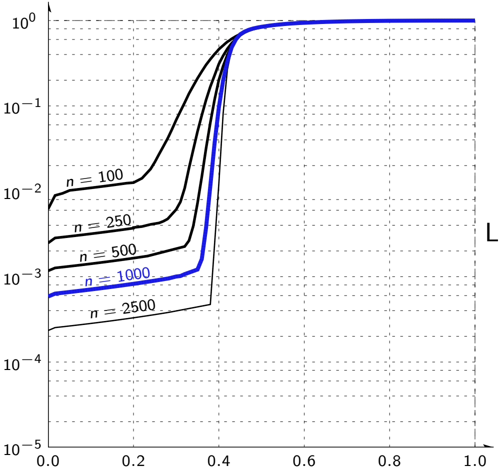


LDPC($n = 500$)

channel noise

Asymptotic Average (Message Passing) Performance

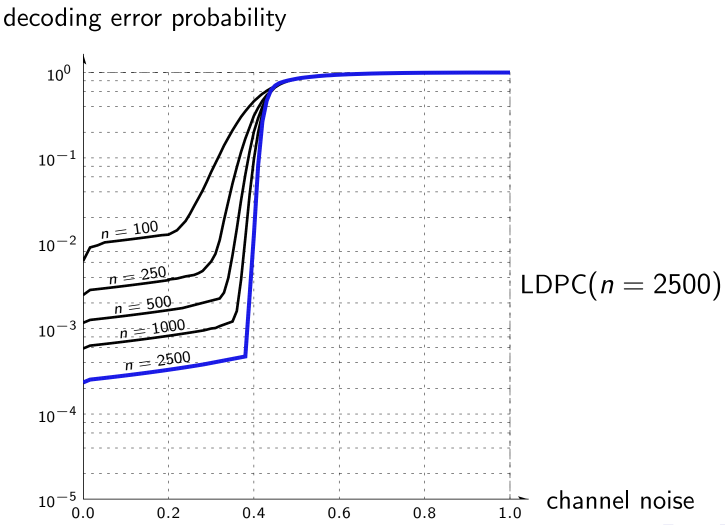
decoding error probability



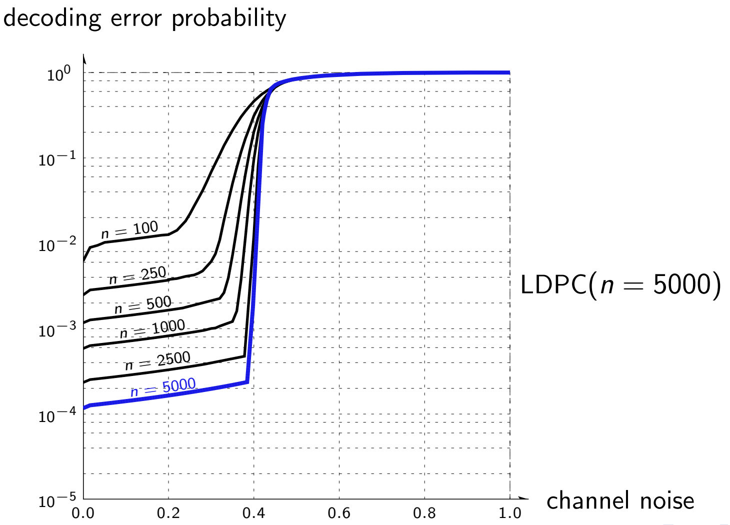
LDPC($n = 1000$)

channel noise

Asymptotic Average (Message Passing) Performance

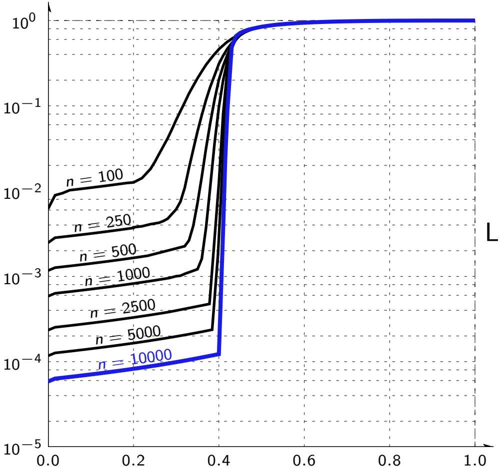


Asymptotic Average (Message Passing) Performance



Asymptotic Average (Message Passing) Performance

decoding error probability

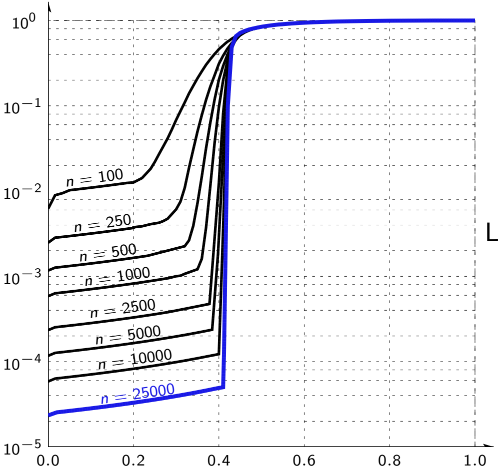


LDPC($n = 10000$)

channel noise

Asymptotic Average (Message Passing) Performance

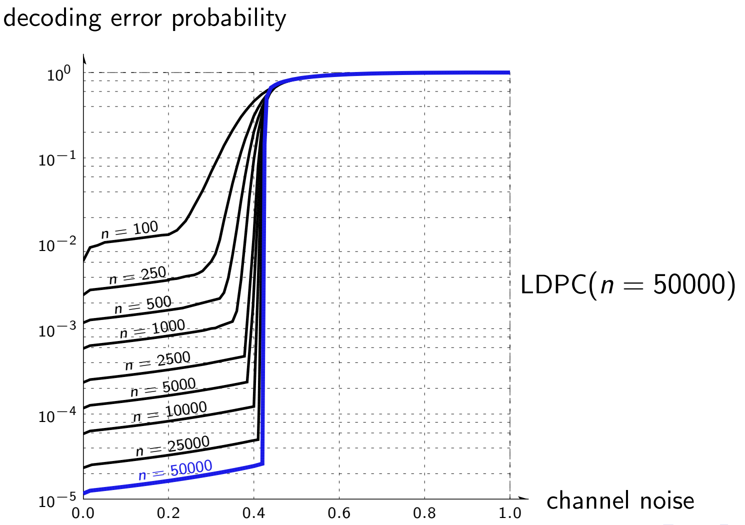
decoding error probability



LDPC($n = 25000$)

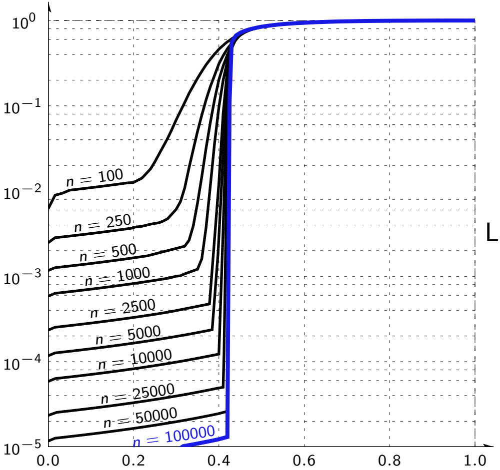
channel noise

Asymptotic Average (Message Passing) Performance



Asymptotic Average (Message Passing) Performance

decoding error probability

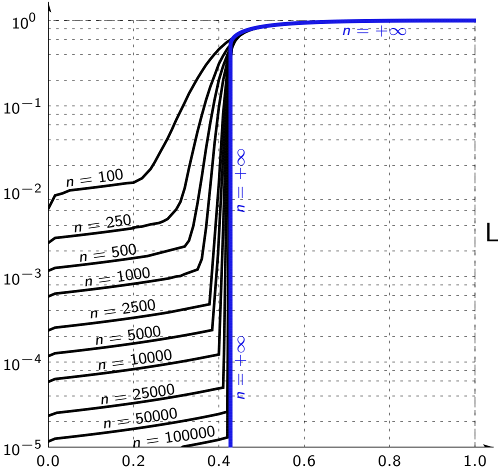


LDPC($n = 100000$)

channel noise

Asymptotic Average (Message Passing) Performance

decoding error probability

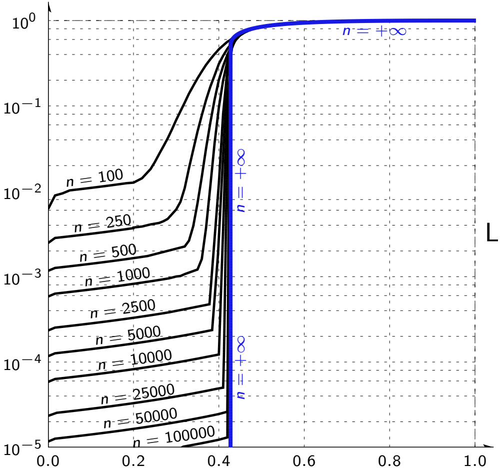


LDPC($n = +\infty$)

channel noise

Asymptotic Average (Message Passing) Performance

decoding error probability

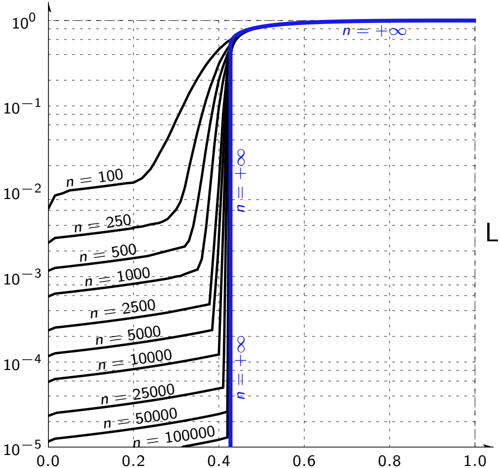


LDPC($n = +\infty$)

channel noise

Asymptotic Average (Message Passing) Performance

decoding error probability

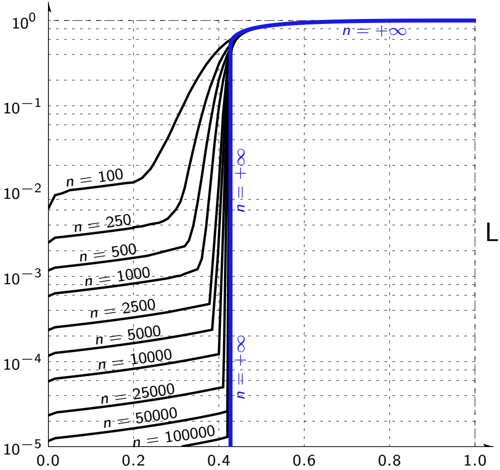


LDPC($n = +\infty$)

channel noise

Asymptotic Average (Message Passing) Performance

decoding error probability

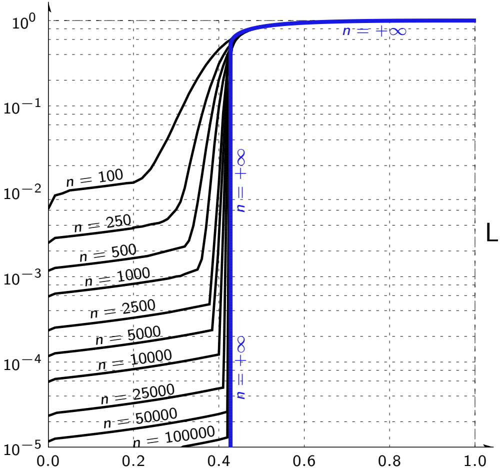


LDPC($n = +\infty$)

channel noise

Asymptotic Average (Message Passing) Performance

decoding error probability

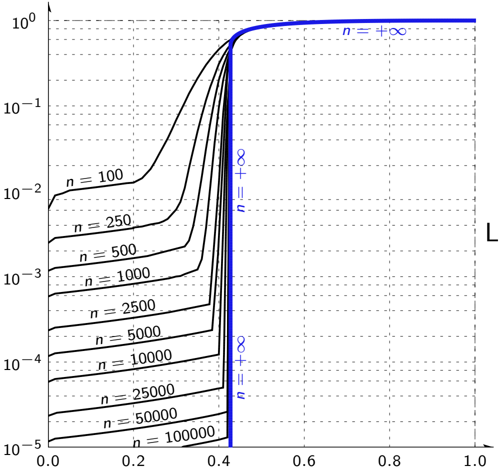


LDPC($n = +\infty$)

channel noise

Asymptotic Average (Message Passing) Performance

decoding error probability

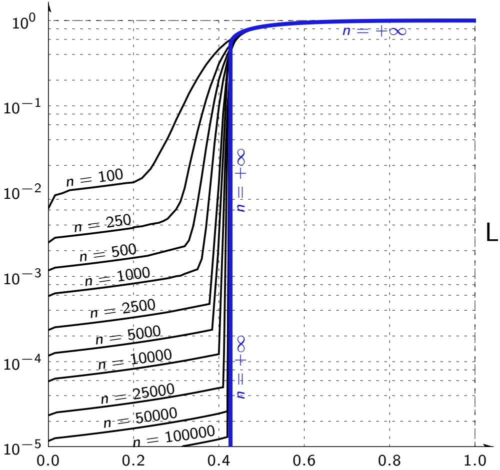


LDPC($n = +\infty$)

channel noise

Asymptotic Average (Message Passing) Performance

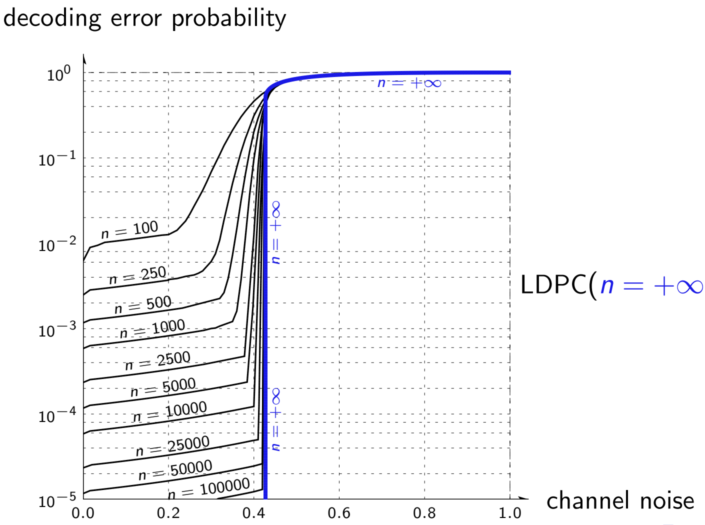
decoding error probability



LDPC($n = +\infty$)

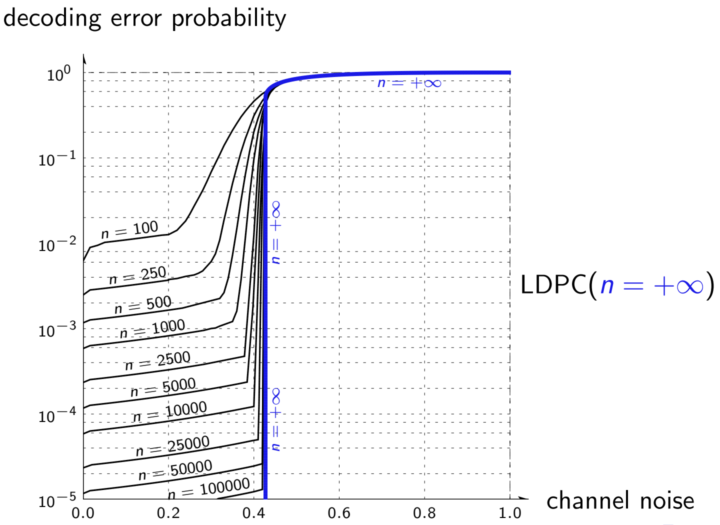
channel noise

Asymptotic Average (Message Passing) Performance



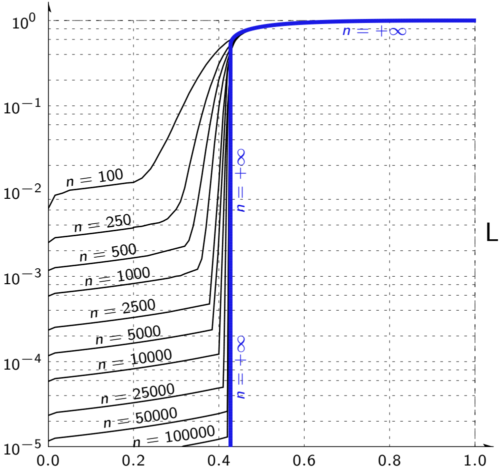
LDPC($n = +\infty$)

Asymptotic Average (Message Passing) Performance



Asymptotic Average (Message Passing) Performance

decoding error probability

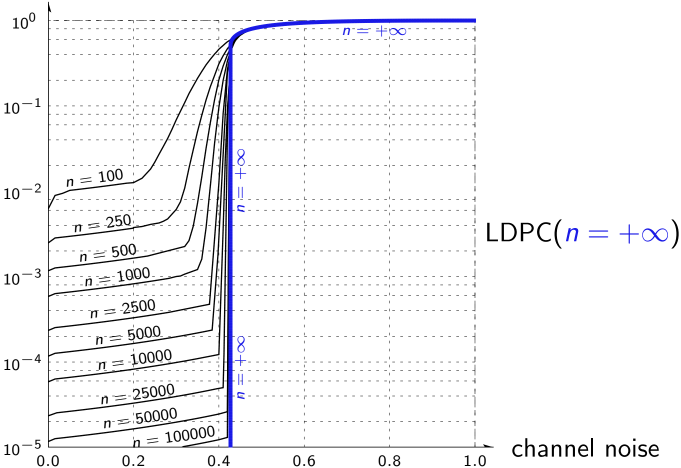


LDPC($n = +\infty$)

channel noise

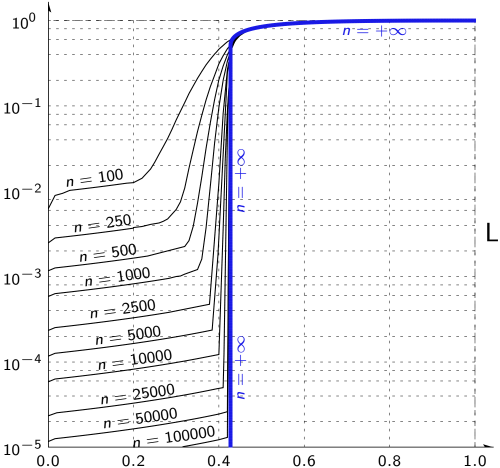
Asymptotic Average (Message Passing) Performance

decoding error probability



Asymptotic Average (Message Passing) Performance

decoding error probability



LDPC($n = +\infty$)

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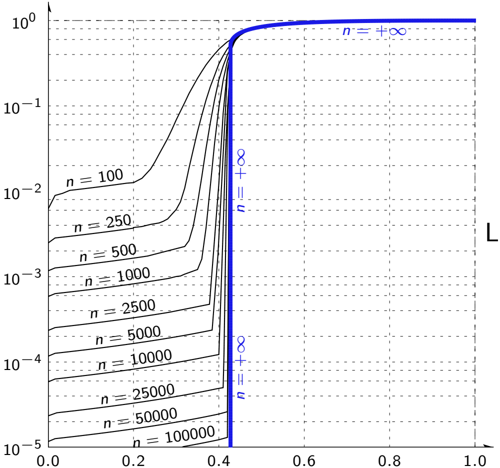


LDPC($n = +\infty$)

channel noise

Constraint satisfaction problems

k-satisfiability

N variables: $\underline{x} = (x_1, x_2, \dots, x_N)$, $x_i \in \{0, 1\}$

M k-clauses

$$(x_1 \vee \overline{x_5} \vee x_7) \wedge (x_5 \vee x_8 \vee \overline{x_9}) \wedge \cdots \wedge (\overline{x_{66}} \vee \overline{x_{21}} \vee \overline{x_{32}})$$

Hereafter $k \geq 4$ (ask me why at the end)

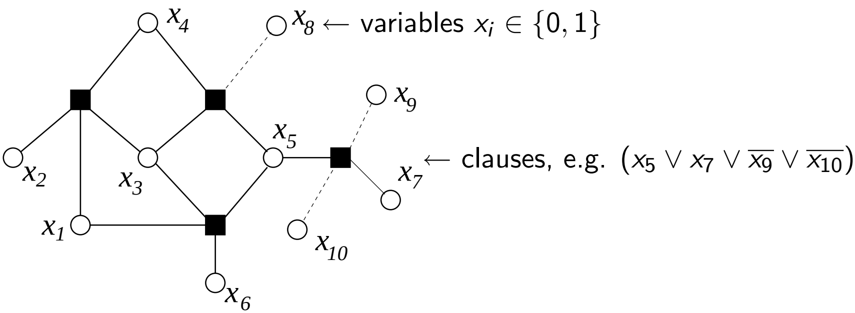
Uniform measure over solutions

$$F = \cdots \wedge \underbrace{(x_{i_1(a)} \vee \overline{x}_{i_2(a)} \vee \cdots \vee x_{i_k(a)})}_{a\text{-th clause}} \wedge \cdots$$

$$\mu(\underline{x}) = \frac{1}{Z} \prod_{a=1}^{M} \psi_a(x_{i_1(a)}, \dots, x_{i_k(a)})$$

$$\psi_a(x_{i_1(a)}, \dots, x_{i_k(a)}) = \begin{cases} 1 & \text{clause } a \text{ satisfied} \\ 0 & \text{otherwise} \end{cases}$$

(Factor) graph representation



Here : $N = 10,\ M = 4$

Distance: $i, j \in \{1, \ldots, N\} \mapsto d(i, j)$

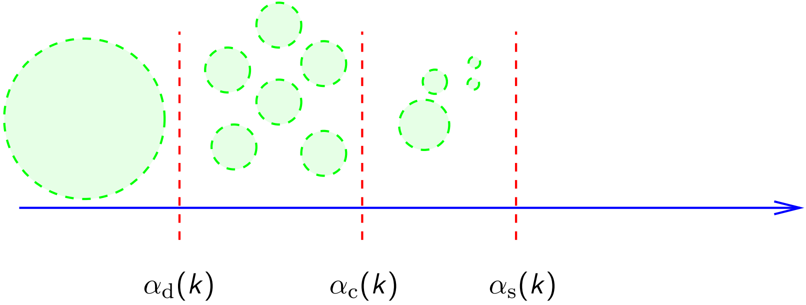
Random k-satisfiability

Each clause is uniformly random among the $2^k \binom{N}{k}$ possible ones.

$N, M \to \infty$ with $\alpha = M/N$ fixed.

Phase transition in the structure of $\mu(\,\cdot\,) \Leftrightarrow$

Set of solutions of F (cavity method):



⇔ Qualitative changes in the correlation strength⇔

$\alpha < \alpha_{\rm d}(k) \Rightarrow$ Weak correlations.

$\alpha_{\rm d}(k) < \alpha < \alpha_{\rm c}(k) \Rightarrow$ Strong *point-to-set* correlations.

$\alpha_{\rm c}(k) < \alpha < \alpha_{\rm s}(k) \Rightarrow$ Strong *point-to-point* correlations.

$\Leftrightarrow$ Performance of message passing algorithms

$\alpha < \alpha_c(k) \Rightarrow$ *Belief propagation* is asymptotically correct.

$\alpha_c(k) < \alpha < \alpha_s(k) \Rightarrow$ *Survey propagation*.

Conclusion 1: Peoples I should have cited/I should thank

Physics : M. Mézard, G. Parisi, R. Monasson, R. Zecchina, F. Ricci-Tersenghi, M. Weigt, G. Biroli, G. Semerjian, N. Sourlas...

CS : D. Achlioptas, D. Gamarnik, E. Mossel, E. Maneva, C. Nair, M. Bayati, D. Weitz, N. Creignou, M. Luby, A. Shokrollahi, A. Sinclair...

Probability : D. Aldous, M. Talagrand, A. Dembo, P. Diaconis, F. Martinelli, Y. Peres, F. Guerra, F. Toninelli...

EE : R. Urbanke, T. Richardson, M. Wainwright, B. Prabhakar, D. Shah, S. Tatikonda, J. Yedidia, D. Forney...

Conclusion 2: If you want to know more about this...

- M. Mézard, A. M., *Upcoming book*
- T. Richardson, R. Urbanke, *Modern Coding Theory*,
  A. M., R. Urbanke, *Les Houches lecture notes*
- 2007 IT Symposium → Statistical Physics tutorial
- 2007 StatPhys symposium → IT Plenary Talk
- google ee374