1. Let $X_1, \ldots, X_n$ be independent rv's with $E(X_i) = 0$ and $|X_i| \leq 1$. Let $\overline{X} = (\sum X_i)/n$ and $\lambda \geq 0$. Show that $\Pr(|\overline{X}| > \lambda/\sqrt{n}) < 2e^{-\lambda^2/2}$.

2. **[No collaboration]** Let $Y_1, \ldots, Y_n$ be independent rv's with $\Pr(Y_i = 1) = p_i, \Pr(Y_i = 0) = 1 - p_i$. Let $Y = \sum Y_i$, $p = (\sum p_i)/n$, and $\beta > 1$. Show that $\Pr(Y > \beta pn) < (e^{\beta-1}\beta^{-\beta})^{pn}$.

3. Let $M(x_1, \ldots, x_n)$ be the *van der Monde* matrix

$$\begin{pmatrix} 1 & x_1 & x_1^2 & \ldots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \ldots & x_2^{n-1} \\ \ldots & & & & \\ 1 & x_n & x_n^2 & \ldots & x_n^{n-1} \end{pmatrix}$$

   Prove that $\mathrm{Det}(M(x_1, \ldots, x_n)) = \prod_{i>j}(x_i - x_j)$.

4. A standard deck of 52 cards has been thoroughly shuffled and placed face down. You turn over cards one at a time until you reach the first ace. What is the probability that the next card is an ace?

5. MU 4.17.

6. Football season is over, so a sports channel shows a fair coin being flipped many times in a row. The audience is allowed to place bets on what happens. For a sequence $w \in \{H, T\}^*$, let X_w be the positive integer random variable denoting the index of the first occurrence of w (as a contiguous subsequence) in the sequence of coin tosses. Determine whether there is probability greater than $1/2$ for either of the following events:

$$X_{HH} < X_{TH}$$

$$X_{HH} > X_{TH}$$

7. **[No collaboration]** MU 1.5

8. MU 1.7

9. MU 1.12

10. MU 2.8