# From Measurements to Models

**3D Photography Class Spring 1997**

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## Geometry for Computer Graphics

**Task**

- describe shapes (geometry)
- what types?

**Requirements**

- expressive
- concise
- easy to manipulate
- easy to process (transmit, render)

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## Geometric Representations

### Parameterized

- general? (manifolds)
- useful
  - properties easy to compute
- problematic
  - properties tied up in parameterization
- geometric tasks: intersection, rendering
- manipulation for modeling

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = g(u, v)$$

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## Parameterized Geometry

### Examples

- line segment $l(t) = p_0 + t(p_1 - p_0) \quad t \in [0,1]$
- triangle

$$T(u,v) = p_0 + u(p_1 - p_0) + v(p_2 - p_0) \quad u \in [0,1]\ v \in [0,1-u]$$

- sphere

$$s(u,v) = \begin{pmatrix} \sin\theta\cos\varphi \\ \cos\theta \\ -\sin\theta\sin\varphi \end{pmatrix} \quad \varphi \in [0,2\pi]\ \theta \in [0,\pi]$$

- piecewise polynomials, i.e., splines

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# Geometric Representations

## Implicit

- elegant $0 = g(x, y, z)$
- solid modeling
  - union, intersection, difference
- general?
- intersection, rendering
- composition, transformation

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# Implicit Geometry

## Examples

- plane $P(x, y, z) = ax + by + cz + d = 0$
  - plane equation $n \cdot p = -d$
- sphere

$$S(x, y, z) = x^2 + y^2 + z^2 - r^2 = 0$$

- isocontour of a probability volume
  - probability of the object being present

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## Different Representations

### Why choose one over another?

- what is fast to display?
- what is easy to manipulate?
- what requires little bandwidth?
- what is easy to match to measured data?
- what supports arbitrary topology?

### Polyhedra

- or higher order versions (subdivision)

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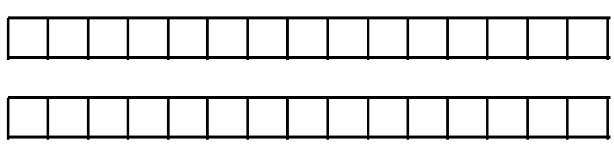
## Representations of Polyhedra

### Indexed face set

- list of vertices
  - and possibly properties associated with them
  - color, normal, texture coordinates, confidence
- list of polygons
  - each a list of indices



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## Representations of Polyhedra

### Indexed face set

- what's easy?
- what's hard?

### Topological data structures

- graph representations
- winged edge

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## From Data to Models

### Given samples of geometry

- build representation
- if generic model available
  - find paramaters through optimization

$$g(x_i, y_i, z_i; \tau_0, \tau_1, \ldots) \stackrel{!}{=} 0 \quad i = 1, \ldots, n$$

- use samples directly for piecewise linear approximation, i.e., polyhedron

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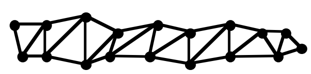
## Building Meshes

### Samples from the scanner

- rectangular array of 3D point measurements
- every(?) camera pixel has associated point and color
- connect up in order to create mesh



- will this work?

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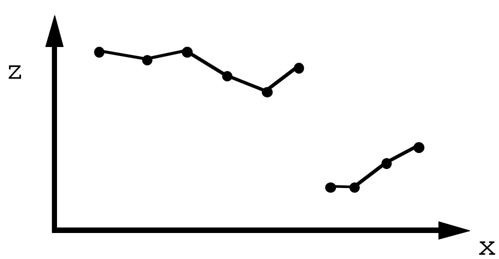
## Building Meshes

### Dealing with holes and discontinuities

- occlusions will lead to discontinuities
- noise will lead to drop outs in data
- only connect if not too distant in Z

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## Fun Facts about Polygons

### Plane equations

- 3 points determine a plane
- how about 4?

### Barycentric coordinates

- points interior to polygon are convex combinations fo vertices

### Ray intersection

- finding closest points

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## Plane Equations

### Triangles

- take cross product of 2 edges

$$\mathbf{n} = (\mathbf{p_1} - \mathbf{p_0}) \mathbf{x} (\mathbf{p_2} - \mathbf{p_0})$$

- numerically sensitive

### What about n-gons?

- can only hope to find best fit plane
- Newell's method

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# Plane Equations

## Newell's method

- given a polygon as list of vertices
- find normal
- best approximation

$$\|\mathbf{n}\| = 2\ \text{Area}(\text{Polygon})$$

$$n_x = \sum_{i=0}^{n-1} (p_z^i + p_z^{i+1})(p_y^i - p_y^{i+1})$$

$$n_y = \sum_{i=0}^{n-1} (p_x^i + p_x^{i+1})(p_z^i - p_z^{i+1})$$

$$n_z = \sum_{i=0}^{n-1} (p_y^i + p_y^{i+1})(p_x^i - p_x^{i+1})$$

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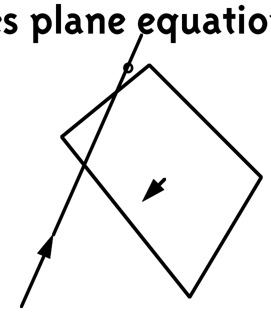
# Intersections

## Ray/Plane

- point along ray which satisfies plane equation
  - plane equation $(\vec{n}, d)$
  - ray equation $\vec{r}(t) = \vec{O} + t\vec{d}$
  - intersection

$$t = \frac{-d - \vec{n} \cdot \vec{O}}{\vec{n} \cdot \vec{d}}$$



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# Intersections

## Ray/Convex Polygon

- intersect support plane
- compute intersection point
- project along dominant direction
  - largest magnitude on plane equation
- solve point in convex polygon problem

```
For All Edges
    if p on negative side
        return 0
return 1
```

$$(\vec{p}-\vec{v}_i)\times(\vec{v}_{i+1}-\vec{v}_i) > 0$$

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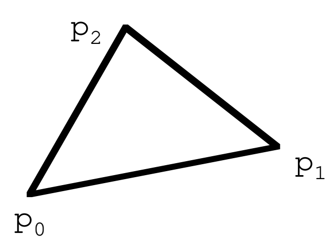
# Barycentric Coordinates

## Convex combinations

- writing interior points as convex combinations of vertices
- useful for interpolation
- e.g., triangle

$$p_I = ap_0 + bp_1 + cp_2 \quad a + b + c = 1$$

$$a = \frac{\det(p_I, p_1, p_2)}{\det(p_0, p_1, p_2)}$$

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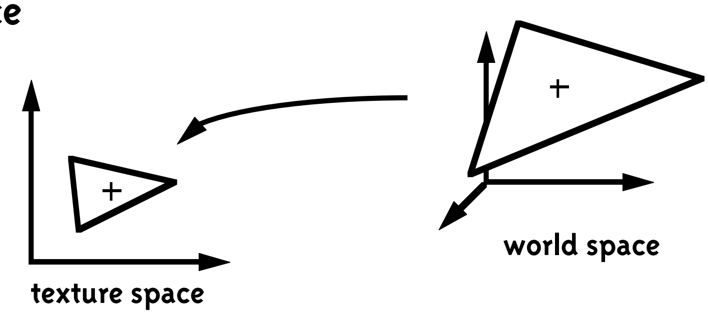
## Texture Maps

### Decorating a surface

- paint a picture onto the surface
- associate texture coordinates with points on surface

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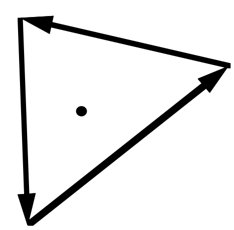
## Sidedness Tests

### Careful with orientation!

- choose canonical orientation of vertex lists
- i.e., CCW

$$\begin{vmatrix} x_{i+1} - x_i & p_x - x_i \\ y_{i+1} - y_i & p_y - y_i \end{vmatrix} > 0$$



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