Reading

Chapters 14 and 15 from Vazirani.

Problems

#1. *(Exercise 8.35 from Cook et al.)* Show that the problem of finding a minimum-cost directed (r, s)-dipath through all the nodes of a digraph G can be reduced to the problem of finding a maxiumum-cardinality common independent set of *three* matroids. (This shows that the latter problem is NP-hard.) [5 points]

#2. *(Exercise 2.14 from Vazirani)* (**MAX k-CUT**) Given an undirected graph $G = (V, E)$ with nonnegative edge costs, and an integer k, find a partition of V into sets $S_1, \ldots, S_k$ so that the total cost of edges running between these sets is maximized.

Give a greedy algorithm for this problem that achieves an approximation factor of $(1 - \frac{1}{k})$. Is the analysis of your algorithm tight? [*Hint:* Look at Exercise 2.1 which addresses the special case of an unweighted graph and $k = 2$. For your convenience, that algorithm is reproduced below:

(a) $A \leftarrow \{v_1\}$ and $B \leftarrow \{v_2\}$

(b) For $v \in V \setminus \{v_1, v_2\}$ do: if $d(v, A) \geq d(v, B)$ then $B \leftarrow B \cup \{v\}$ else $A \leftarrow A \cup \{v\}$

(c) Output A and B

Here v_1 and v_2 are arbitrary vertices of G and for $A \subset V$, the quantity $d(v, A)$ denotes the number of edges from vertex v to set A.] [5 points]

#3. *(Exercise 12.4 from Vazirani)*Consider the LP-relaxation and dual LP for the set cover problem. Let $\mathbf{x}$ and $\mathbf{y}$ be primal and dual feasible solutions, respectively, and assume that they satisfy complementary slackness conditions. Show that the dual "pays" for the primal exactly via a "local paying mechanism" as follows: If each element e pays $y_e x_S$ to each set S containing e, the the amount collected by each set S is precisely $c(S)x_S$. Hence show that $\mathbf{x}$ and $\mathbf{y}$ have the same objective function values. [5 points]