Reading

Review basic approximation algorithms for Vertex Cover, Set Cover, Euclidean TSP. Read Chapters 8 and 11 from Vazirani.

Problems

#1. Use Theorem 8.4 from Cook et al [Greedy algorithm produces an optimal solution of

$$\begin{aligned} \max \sum_{e \in S} c_e x_e & \\ \text{subject to } \mathbf{x}(A) \leq r(A); & \qquad A \subseteq S \\ x_e \geq 0; & \qquad e \in S.] \end{aligned}$$

and the separating hyperplane lemma to prove Theorem 8.5: Let $(S, \mathcal{I})$ be a matroid with rank function r. The convex hull of characteristic vectors of independent sets is $\{\mathbf{x} \in \mathbb{R}^S : \mathbf{x} \geq \mathbf{0}, \mathbf{x}(A) \leq r(A) \text{ for all } A \subset S\}$. [5 points]

#2. *(Graph painting)* Devise an efficient algorithm for the following problem: Given a graph G, paint each of its edges one of Red, White, or Blue in such a manner that none of its cycles are monochromatic (i.e., all its edges are of the same color). [5 points]

#3. *(Exercise 8.43 from Cook et al)* Show that there does not exist a set J that is the set of edges of a spanning tree in both of the graphs of Figure 1.

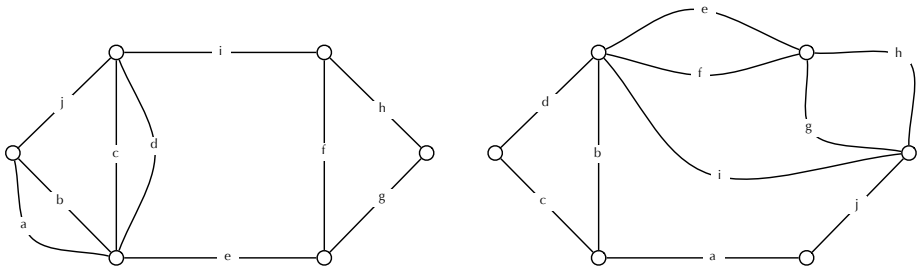

Figure 1: Common spanning tree?

[5 points]