Exercise 3, Part 2

For this part of Homework 5, you may find it easier to work with the following definition of Ackermann's function. We first define a function $A_n(x)$ as follows.

$$\begin{array}{rcl} A_0(x) & = & x+1 \\ A_{n+1}(x) & = & A_n^{(x+1)}(1) = \underbrace{A_n(A_n(\ldots A_n}_{x+1 \text{ times}}(1)\ldots)) \end{array}$$

For each fixed n, $A_n(x)$ is primitive recursive. (You don't have to explicitly show this, but you should be able to convince yourself of this fairly easily.) We can then define Ackermann's function as a 2-ary function $A(n, x) = A_n(x)$. This definition is equivalent to the more prevelant definition of Ackermann's function given below.

$$\begin{array}{rcl} A(0, x) & = & x+1 \\ A(n+1, 0) & = & A(n, 1) \\ A(n+1, x+1) & = & A(n, A(n+1, x)) \end{array}$$

(This last definition is the one given in Kozen.)

Important: For Part 1, you should use the definition of Ackermann's function given in lecture (and on the set).