

Exercise 1. Context-free grammars

The grammar $e ::= e + e \mid e * e \mid (e) \mid \text{NUMBER}$ generates the set of arithmetic expressions with $+$, $*$, parentheses, and numbers. The grammar is ambiguous, since, for example, $\text{NUMBER} + \text{NUMBER} * \text{NUMBER}$ has multiple derivations.

Construct an equivalent unambiguous grammar.

The idea is to impose an ordering so that for any sentence in the language there is only one derivation. We do that by splitting the grammar into several levels.

$$\begin{aligned} S &::= T \mid T + S \\ T &::= N \mid N * T \\ N &::= \text{NUMBER} \mid (S) \end{aligned}$$

Exercise 2. Pushdown-automata

Construct a PDA equivalent to the following grammar (the capital letters represent terminals).

$$\begin{aligned} s &::= Aaa \\ a &::= As \mid Bs \mid A \end{aligned}$$

This grammar is in Greibach Normal Form (all productions are made of one terminal followed by zero or more non-terminals), so we can use the recipe given in class (Oct. 17, pg. 22) to convert it to an NPDA that accepts by empty stack.

$$M = (\{q\}, \{A, B\}, \{s, a\}, \delta, q, s, \{\})$$

where:

$$\begin{aligned} \delta(q, A, s) &= \{(q, aa)\} \\ \delta(q, A, a) &= \{(q, s), (q, \epsilon)\} \\ \delta(q, B, a) &= \{(q, s)\} \end{aligned}$$