CS 175 WRITTEN HOMEWORK 1: DUE 4/12/2005

April 6, 2005

1. Polynomial interpolation

(10pts) Recall that the Lagrange interpolation formula which interpolates a given function f at the points $x_0,\ x_1, \ldots,\ x_n$ is given by

$$p(x) = \sum_{i=0}^{n} l_i(x) f(x_i),$$

where l_i are given by

$$l_i(x) = \prod_{0 \le k \le n, k \ne i} \left(\frac{x - x_k}{x_i - x_k} \right)$$

The Newton's Divided Difference Polynomial which gives the same interpolating polynomial has an alternative form

$$p(x) = f(x_0) + (x - x_0) f[x_0, x_1] + (x - x_0)(x - x_1) f[x_0, x_1, x_2] + \ldots$$
$$+ \prod_{i=0}^{n-1} (x - x_i) f[x_0, x_1, \ldots, x_n]$$

where divided difference can be calculated recursively by

$$f[x_0, x_1, \ldots, x_n] = \frac{f[x_1, x_2, \ldots, x_n] - f[x_0, x_1, \ldots, x_{n-1}]}{x_n - x_0}$$

and $f[x_i] = f(x_i)$.

Compare the two methods above and point out which one is more easy to evaluate when extra interpolating points are added. Explain why.

2. Affine maps

(20pts) Prove: A function $f : \mathbf{R}^n \to \mathbf{R}^m$ is an affine map if and only if f can be represented as a linear map plus a translation, e.g. $f(x) = L(x) + b, x \in \mathbf{R}^n, b \in \mathbf{R}^m$.

3. Polar forms

(15pts) Let $f(u_1, u_2, u_3)$ be a polar form. Given that

$$f(0, 0, 1) = a, f(1, 1, 0) = b, f(0, 1, 2) = c, f(3, 2, 0) = d,$$

find those of the following values that are possible to express in terms of a, b, c, d and show how they can be computed (use diagrams):

$$f(2, 0, 0), f(2, 3, 4), f(1, 1, 1), f(0, 1, 3), f(0, 4, 1).$$

4. Blossoming

(1) (10pts) Find the blossoms $f_{(2)}(u_1, u_2)$ and $f_{(3)}(u_1, u_2, u_3)$ of $F(u) = 7 - 2u^1 + 8u^2$.
(2) (15pts) Show that in the general case

$$f_{(2)}(u_1, u_2) + f_{(2)}(u_2, u_3) + f_{(2)}(u_3, u_1) = 3f_{(3)}(u_1, u_2, u_3).$$

This is called *degree raising.*

5. Differencing

(1) (10pts) Show that $\Delta[\tau_1, \tau_2]f = \Delta[\tau_2, \tau_1]f$.
(2) (25pts) Prove the following statement:

Lemma 1. *Let f be a polar n-form, and $0 \leq p \leq n$. Fix $u \in \mathbf{R}$. If for all $q = 0, \ldots, p$ and for all $\tau_1, \ldots \tau_p \in \mathbf{R}$ we have*

$$\Delta[\tau_1, \ldots, \tau_q]f(\underbrace{u, \ldots, u}_{n-q}) = 0;$$

then for all $v_1, \ldots, v_p \in \mathbf{R}$ the following equality holds

$$f(\underbrace{u, \ldots, u}_{n-p}, v_1, \ldots, v_p) = 0.$$