CS 286a | **HW 2: Learning to schedule** | **Guru: Adam Wierman**

Assigned: 2/2/09 | Due: 2/13/09 noon

I encourage you to discuss these problems with others, but you need to write up the actual solutions alone. At the top of your homework sheet, please list all the people with whom you discussed. Crediting help from other classmates will not take away any credit from you. Also, please limit your use of the web as it is difficult to develop good problems, and so enough search will probably lead you to find the solutions online.

1 Prep for student presentations [0 points]

I expect everyone to read at least the introduction of the papers being presented before the presentations.

2 Learning from our analysis [35 points]

In order to develop some intuition for all the messy formulas we've derived for $E[T]$ under various scheduling policies, let's do some numerics.

For this problem, consider an M/GI/1 queue with two different job size distributions: a Weibull with $\alpha = 1, 0.5$. In both cases fix the mean equal to 1. Make two plots for each job size distribution. (Recall that $E[S]$ is the mean slowdown.)

- $(1-\rho)E[T]$ versus ρ
- $(1-\rho)E[S]$ versus ρ

On each plot, include one line for each of FCFS, PS, SJF, FB, and SRPT. *Turn in your code, the plots, and a discussion of the plots.* (Note: I find Mathematica to be the easiest tool for this sort of calculation.)

Question: Why did I choose $(1-\rho)E[T]$ instead of $E[T]$?

3 How unfair are SRPT & FB? [15 points]

As we discussed in class, it is common to use the mean response time for a job of size x, $E[T(x)]$, to contrast the "fairness" of performance for different job sizes. In this problem, you will prove some of the initial results about fairness that started me thinking about the topic!

For this question, we'll treat PS as a "fair" policy and then try to understand how SRPT compares to PS.

(a) Prove that the largest job sizes are not treated any worse under SRPT than they are under PS, i.e.,

$$\lim_{x\to\infty} \frac{E[T(x)]^{SRPT}}{x} = \lim_{x\to\infty} \frac{E[T(x)]^{FB}}{x} = \lim_{x\to\infty} \frac{E[T(x)]^{PS}}{x}$$

(b) Prove that if $\rho \leq 1/2$ then $E[T(x)]^{SRPT} \leq E[T(x)]^{PS}$ for all x. Does a similar result hold for FB?

4 Busy periods, again [10 points]

Prove that the length of an M/GI/1 busy period is stochastically increasing in both λ and X. That is, prove that if $\lambda_1 \leq \lambda_2$ and $X_1 \leq_{st} X_2$, then $B_1 \leq_{st} B_2$, where B_i is the busy period in an M/GI/1 queue having arrival rate λ_i and job sizes X_i.

5 When analysis fails...simulate! [40 points]

Last homework you simulated random variables, this time you'll use that tool in order to build your first simulator of a queue – specifically an M/GI/1 queue with FCFS scheduling. Feel free to use any language/tool in order to build your simulator.

Your goal will be to measure $E[T]$ and you should accomplish this by averaging many samples. Let one run consist of running the system from an empty state and then recording the response time of the 50,000th job. Your final average should be of at least 1000 runs.

Question: Why did I set up a run in this way?

Run your simulator for $\rho = (.2, .4, .6, .8, .9, .95)$ with a Weibull $\alpha = 0.5$ mean 1 job size distribution. Then, make a plot showing the analytic $E[T]$ (problem 1) along with the data from the simulator. *Turn this plot, your code, and a discussion of the results.*

Extra Credit: Repeat this problem for an M/GI/1 SRPT queue.