Homework CS/Ec 241a winter 2008

John Ledyard

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There is one item to distribute to N people. Individual i has a value for the item $v^i \in [a^i, b^i]$. An allocation is a vector $x \in \{0,1\}^N$ where $x^i = 1$ iff i gets the item. An outcome is $p \in \triangle(X)$ where $\triangle(X)$ is the simplex which is $\{p \in R^N | \sum p^i = 1 \text{ and } p^i \geq 0 \quad \forall i\}$. There is no transferable good, no medium of exchange. So the (expected) payoff to player i is $v^i p^i$.

You are a mechanism designer and the performance standard is expected aggregate utility. That is you want to get the largest value possible for $W(v,p) = \sum_i v^i p^i$.

Question 1: What is the set of all Direct Revelation Dominant Strategy Incentive Compatible mechanisms?

Question 2: Describe one mechanism that is Ex Post Incentive Efficient.

Question 3: Suppose you have a prior over the v, given by F(v). What dominant strategy mechanism yields the highest expected value? That is, what dominant strategy mechanism maximizes $\int \sum_i p^i(v) v^i dF(v)$?

Question 4: Suppose we relax incentive compatibility to Bayesian Incentive compatibility. Assume the common prior is $F(v) = F^1(v^1)...F^N(v^N)$ so the v^i are independently distributed. What is the set of all Direct Revelation Bayesian Incentive Compatible mechanisms?

Question 5: Describe all the Ex Ante BIC mechanisms. Describe all the Interim Efficient BIC mechanisms.

Question 6: Now assume there is a transferable good so that each i's payoff is now $v^i x^i - t^i$ where t^i is the amount of the transferable good that i receives. We require that the transferable good be neither created nor destroyed. That is, we require that $\sum t^i = 0$. An outcome is now (p,t) where p^i is the probability that i gets the good and t^i is i's (expected) payment. Go back and re-answer questions 1-5 for this new environment.