CS184a:
Computer Architecture (Structure and Organization)

Day 16: February 14, 2003

Interconnect 6: MoT

Previously

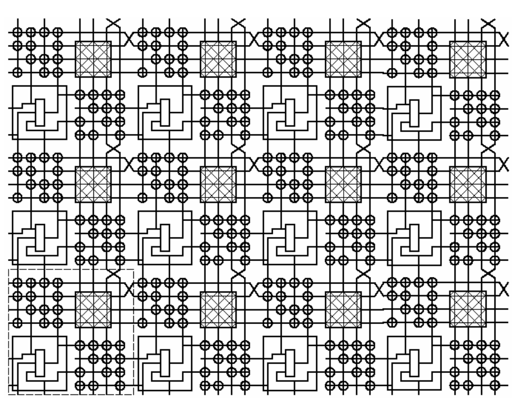
- HSRA/BFT – natural hierarchical network
  - Switches scale O(N)
- Mesh – natural 2D network
  - Switches scale $\Omega(N^{p+0.5})$

Today

- Good Mesh properties
- HSRA vs. Mesh
- MoT
- Grand unified network theory ☺
  – MoT vs. HSRA
  – MoT vs. Mesh

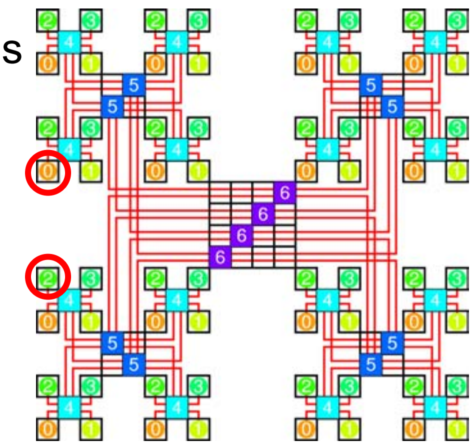
Mesh

1. Wire delay can be Manhattan Distance
2. Network provides Manhattan Distance route from source to sink



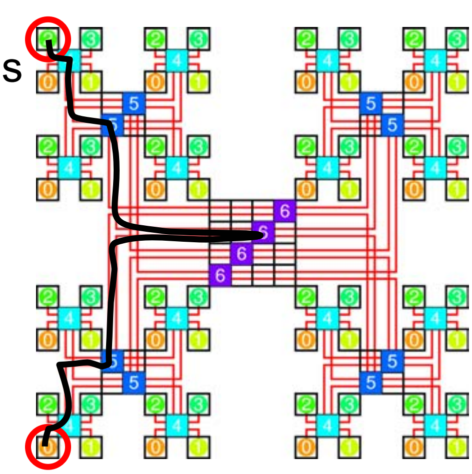
HSRA/BFT

- Physical locality does not imply logical closeness



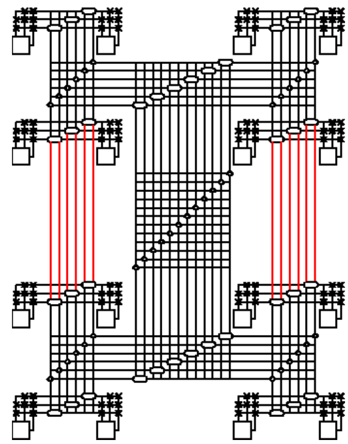
HSRA/BFT

- Physical locality does not imply logical closeness
- May have to route twice the Manhattan distance

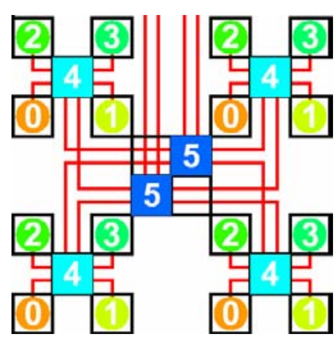


Tree Shortcuts

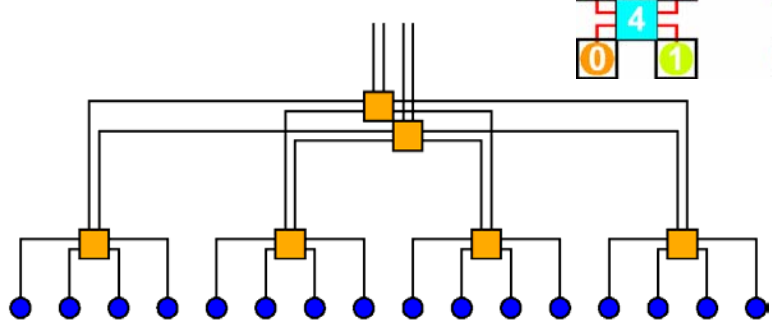
- Add to make physically local things also logically local
- Now wire delay always proportional to Manhattan distance
- May still be 2× longer wires



BFT/HSRA ~ 1D



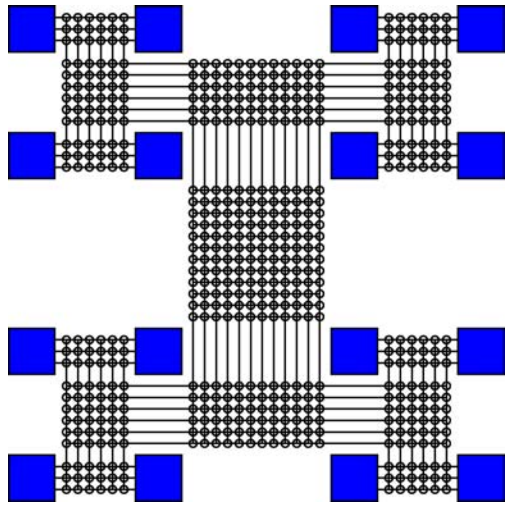
- Essentially one-dimensional tree
  – Laid out well in 2D



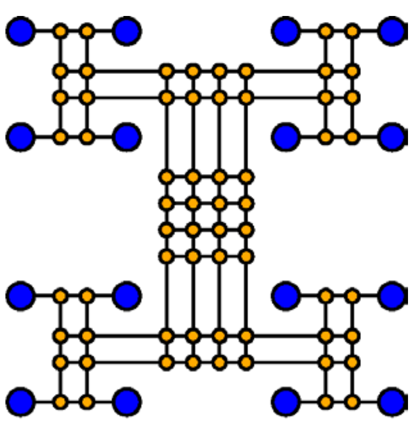
Consider Full Population Tree

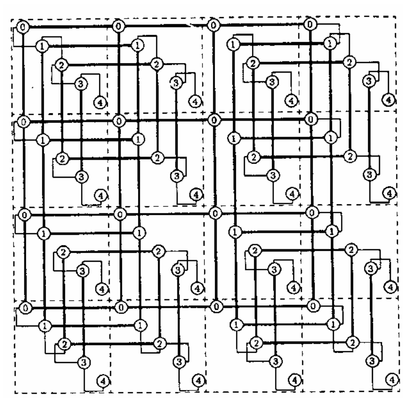
ToM

Tree
of
Meshes

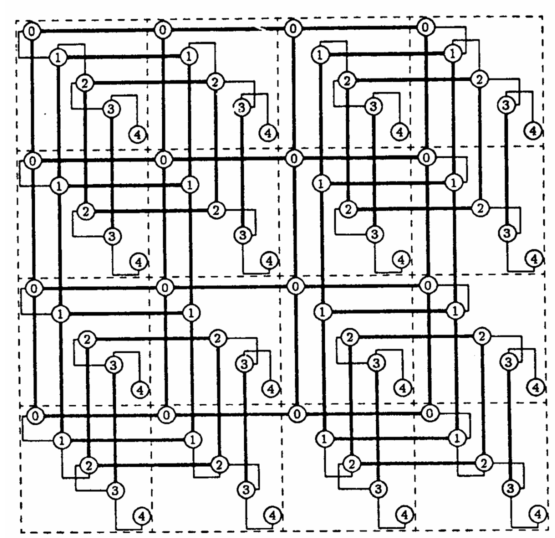


Can Fold Up





Gives Uniform Channels

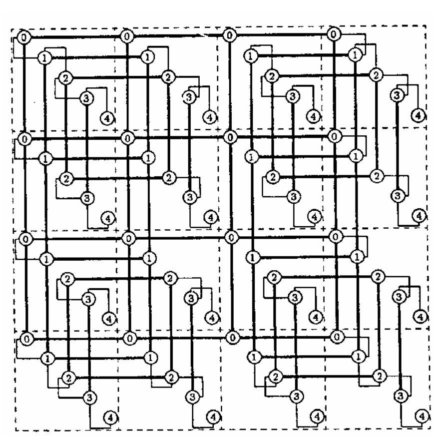


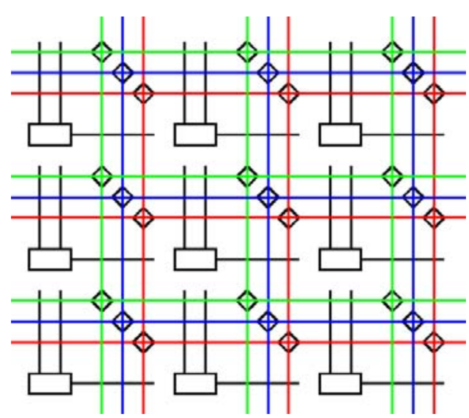
Works nicely
p=0.5

Channels log(N)

[Greenberg and Leiserson, *Appl. Math Lett.* v1n2p171, 1988]

Gives Uniform Channels





(and add shortcuts)

How wide are channels?

$$W = \left\lceil \frac{w(N)+w(N/2)}{\sqrt{N}} \right\rceil + \left\lceil \frac{w(N/4)+w(N/8)}{\sqrt{\frac{N}{4}}} \right\rceil + \ldots$$

$$w(N) = c\,N^{p}$$

$$W = \left(\frac{c\,N^{p}}{\sqrt{N}}\right) \times \left(1 + 2^{-p} + \left(1+2^{-p}\right) \times 2 \times 2^{-2p} + \ldots\right)$$

How wide are channels?

$$W = \left(\frac{c\,N^{p}}{\sqrt{N}}\right) \times \left(1 + 2^{-p} + \left(1+2^{-p}\right) \times 2 \times 2^{-2p} + \ldots\right)$$

$$W = \left(c\,N^{p-0.5}\right) \times \left(1+2^{-p}\right) \times \left(1 + 2^{1-2p} + 2^{2\times(1-2p)} + \ldots\right)$$

$$W = \left(c\,N^{p-0.5}\right) \times \left(1+2^{-p}\right) \times \left(\frac{1}{1-2^{1-2p}}\right)$$

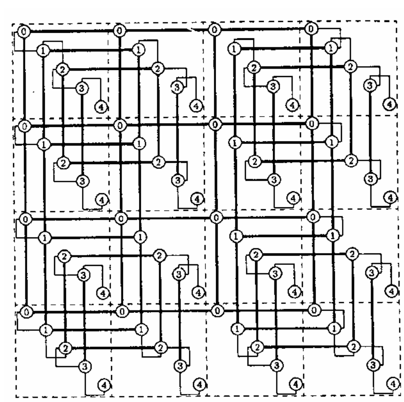
How wide are channels?

$$W = \left(c\,N^{p-0.5}\right) \times \left(\frac{1+2^{-p}}{1-2^{1-2p}}\right)$$

- A constant factor wider than lower bound!
- P=2/3 → ~8
- P=3/4 → ~5.5

Implications

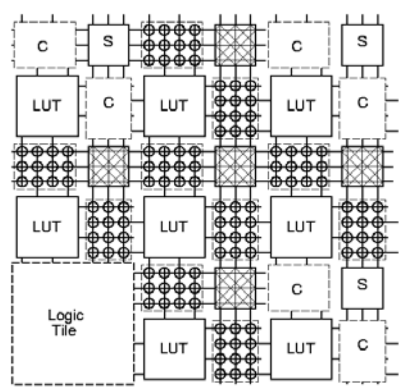
- Tree never requires more than constant factor more wires than mesh
  - Even w/ the non-minimal length routes
  - Even w/out shortcuts
- Mesh global route upper bound channel width is $O(N^{p-0.5})$
  - Can always use fold-squash tree as the route



MoT

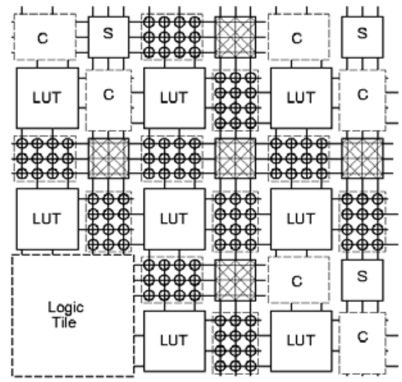
Recall: Mesh Switches

- Switches per switchbox:
  - $6w/L_{seg}$
- Switches into network:
  - $(K+1)\ w$
- Switches per PE:
  - $6w/L_{seg} + Fc \times (K+1)\ w$
  - $w = cN^{p-0.5}$
  - Total $\propto N^{p-0.5}$
- Total Switches: $N^*(Sw/PE) \propto N^{p+0.5} > N$



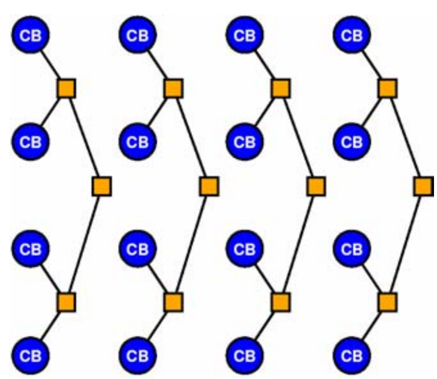
Recall: Mesh Switches

- Switches per PE:
  - $6w/L_{seg} + Fc \times (K+1)\, w$
  - $w = cN^{p-0.5}$
  - Total $\propto N^{p-0.5}$
- Not change for
  - Any constant Fc
  - Any constant L_{seg}



Mesh of Trees

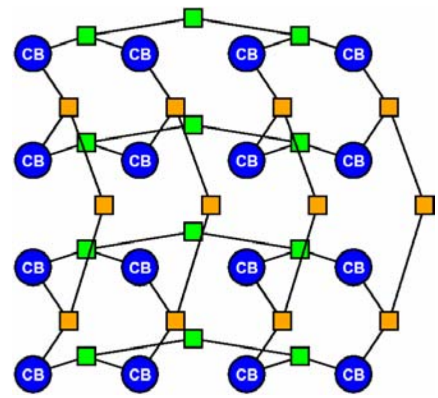
- Hierarchical Mesh
- Build Tree in each column



[Leighton/FOCS 1981]

Mesh of Trees

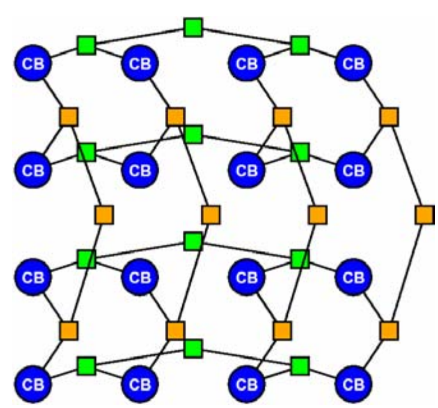
- Hierarchical Mesh
- Build Tree in each column
- …and each row



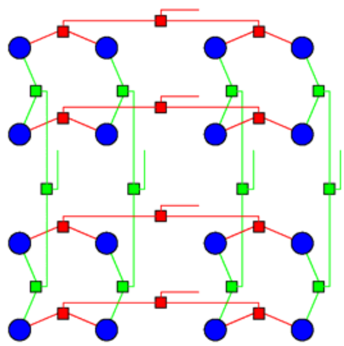
[Leighton/FOCS 1981]

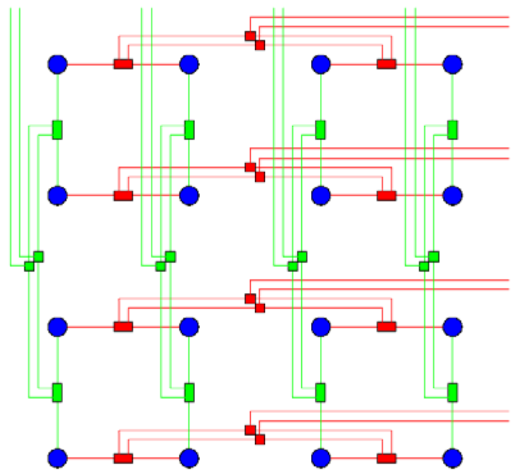
Mesh of Trees

- More natural 2D structure
- Maybe match 2D structure better?
  – Don't have to route out of way



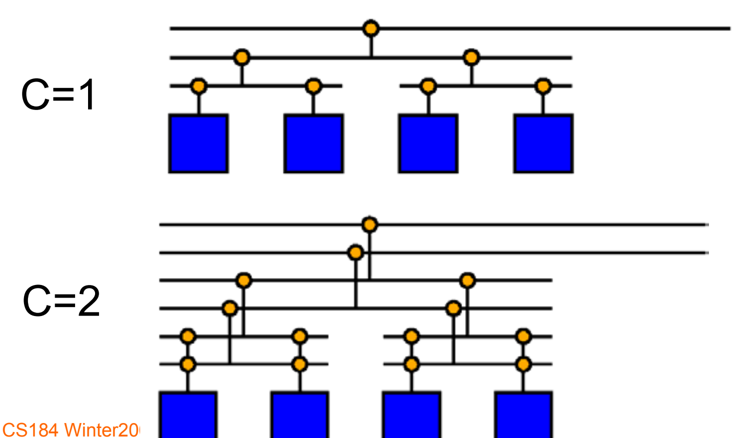
Support P





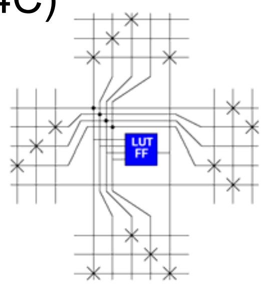
MoT Parameterization

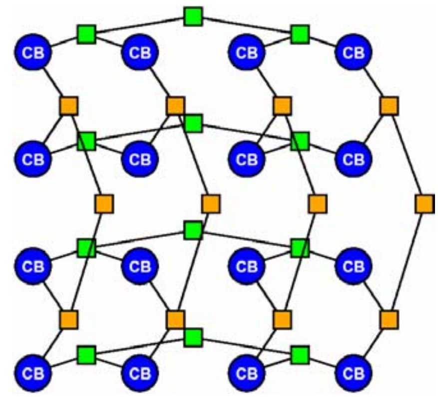
- Support C with additional trees



Mesh of Trees

- Logic Blocks
  - Only connect at leaves of tree
- Connect to the C trees (4C)



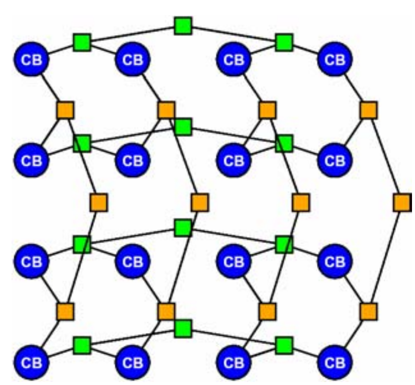


Switches

- Total Tree switches
  - 2 C (switches/tree)
- Sw/Tree:

$$\left(\frac{N}{2}\right)\times\left(1+\frac{2^{p-0.5}}{2}+\left(\frac{2^{p-0.5}}{2}\right)^2+\ldots\right)$$

$$\left(\frac{N}{2}\right)\times\left(\frac{1}{1-2^{p-1.5}}\right)$$



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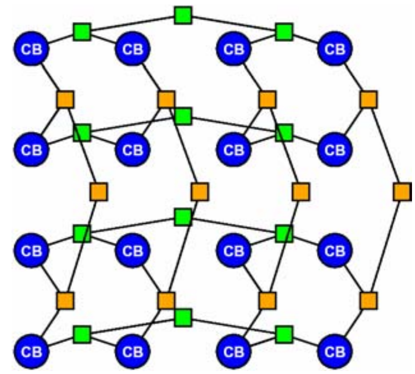
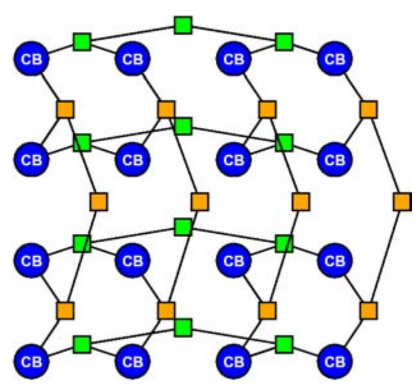


$$TreeSwitches=\left(\frac{C\times N}{1-2^{p-1.5}}\right)=\mathrm{O}(N)$$

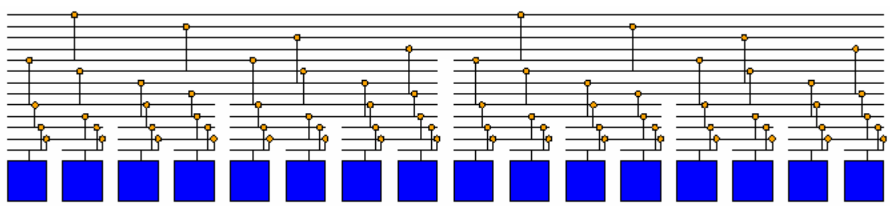
Switches

- Only connect to leaves of tree
- C×(K+1) switches per leaf
- Total switches
  - Leaf + Tree
  - O(N)



Wires

- **Design:** $O(N^p)$ in top level
- Total wire width of channels: $O(N^p)$
  - Another geometric sum
- No detail route guarantee (at present)

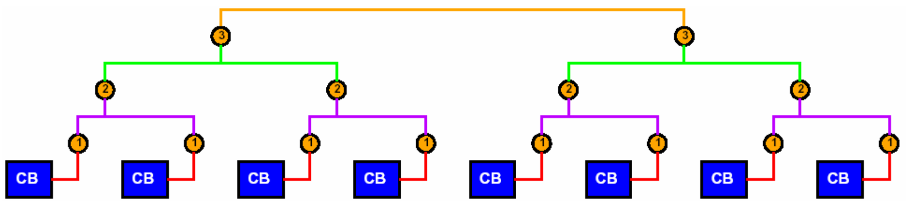


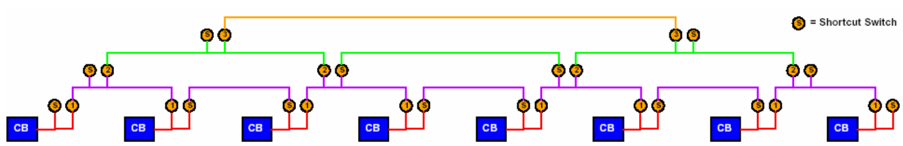
Empirical Results

- **Benchmark:** Toronto 20
- Compare to L_{seg}=1, L_{seg}=4
  - CLMA ~ 8K LUTs
    - Mesh(L_{seg}=4): w=14 → 122 switches
    - MoT(p=0.67): C=4 → 89 switches
  - Benchmark wide: 10% less
    - CLMA largest
    - Asymptotic advantage

Shortcuts

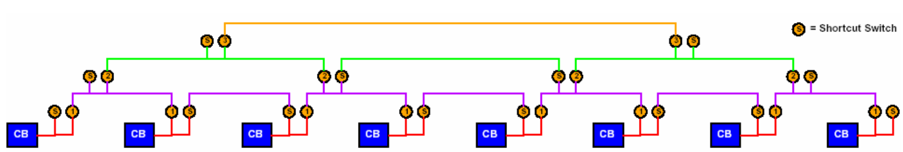
- Strict Tree
  – Same problem with physically far, logically close





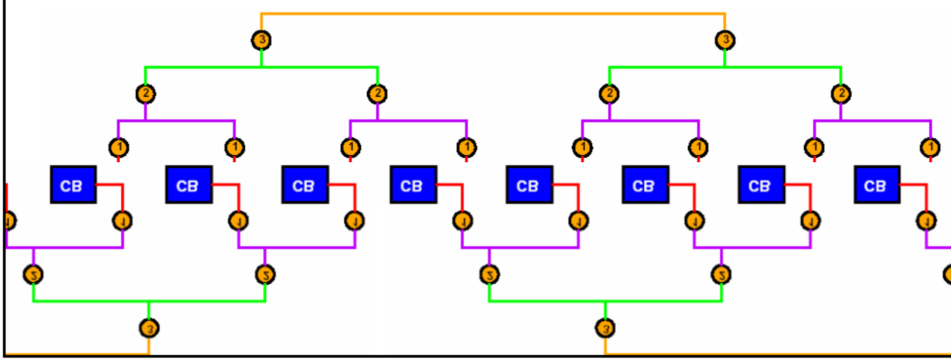
Shortcuts

- Empirical
  – Shortcuts reduce C
  – But net increase in total switches



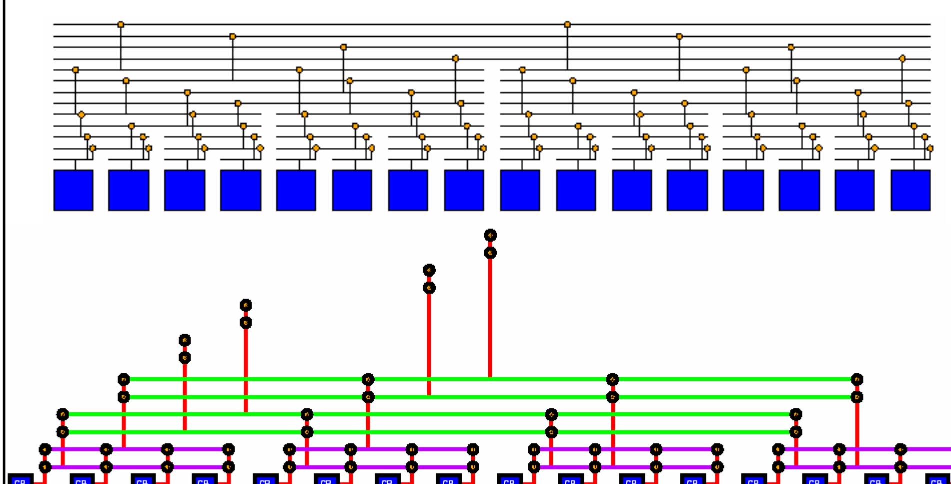
Staggering

- With multiple Trees
  - Offset relative to each other
  - Avoids worst-case discrete breaks
  - One reason don't benefit from shortcuts



Flattening

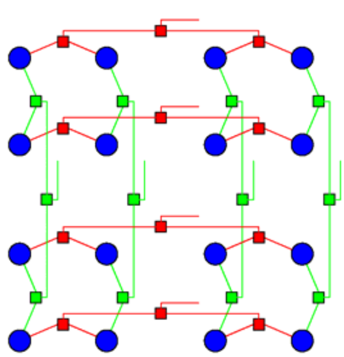
- Can use arity other than two

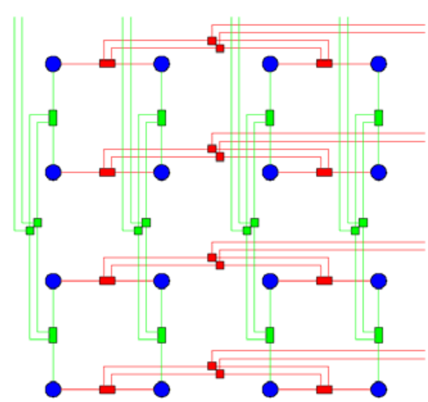


MoT Parameters

- Shortcuts
- Staggering
- Corner Turns
- Arity
- Flattening

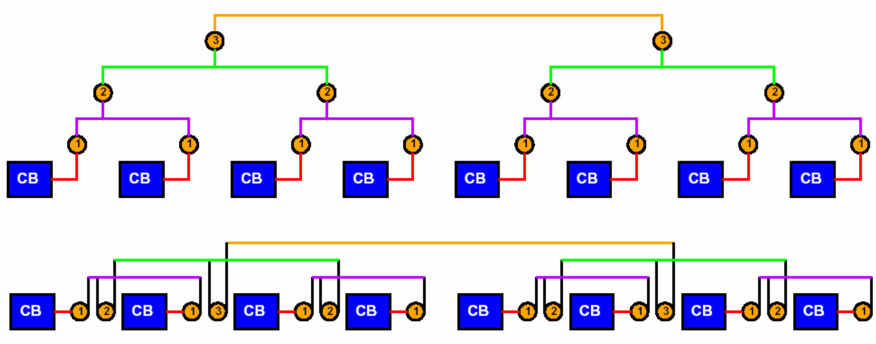
MoT Layout



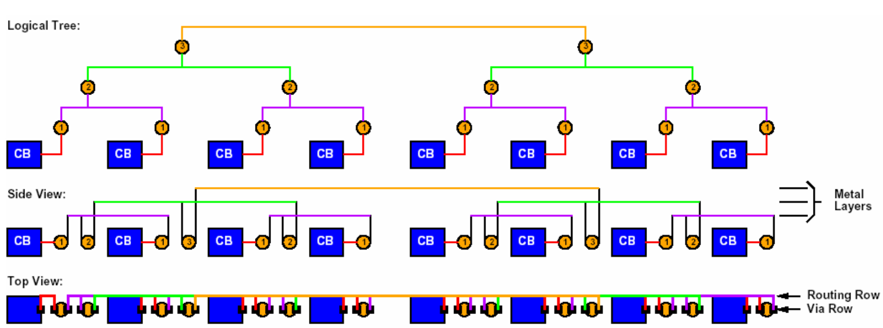


Main issue is layout 1D trees in multilayer metal

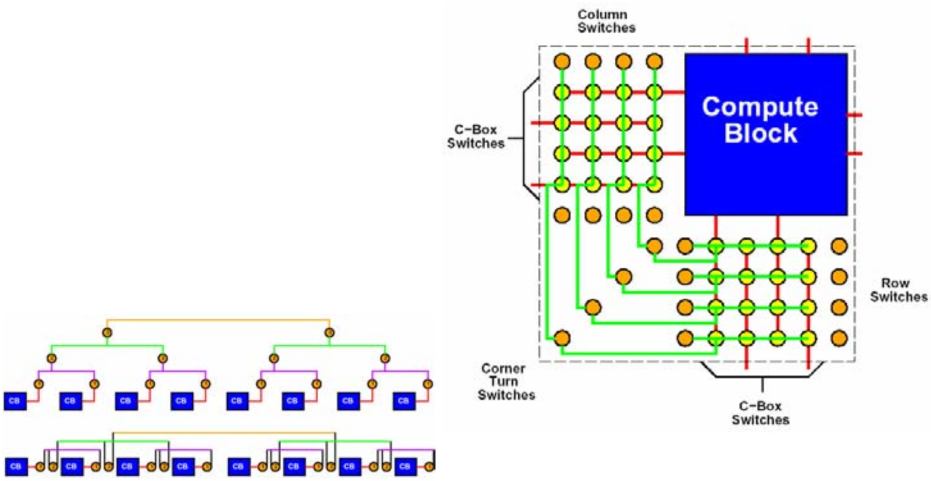
Row/Column Layout



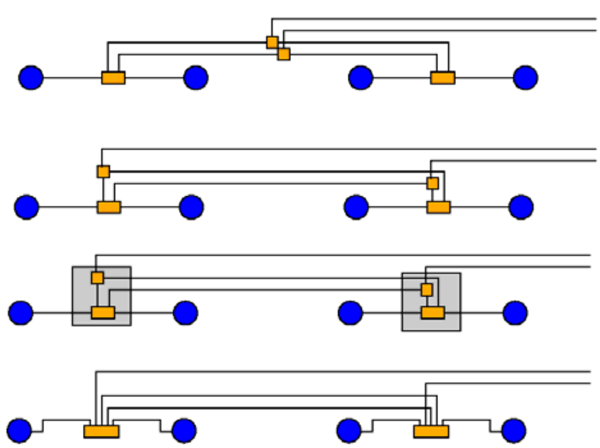
Row/Column Layout



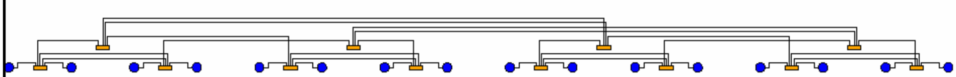
Composite Logic Block Tile



P=0.75 Row/Column Layout



P=0.75 Row/Column Layout



MoT Layout

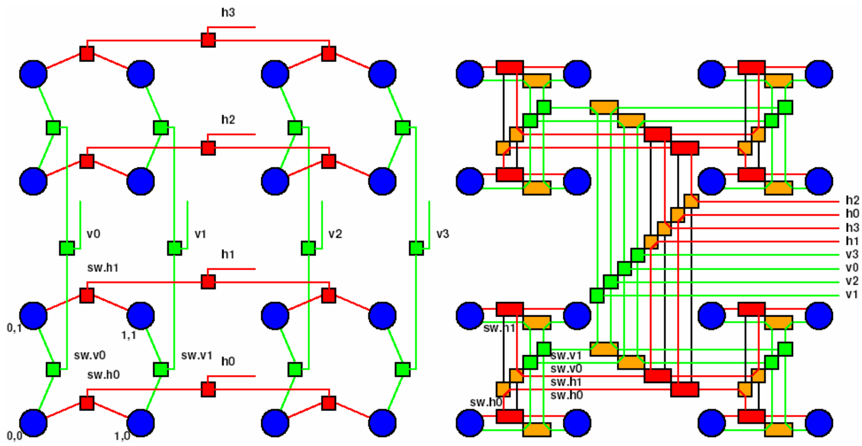
- Easily laid out in Multiple metal layers
  - Minimal $O(N^{p-0.5})$ layers
- Contain constant switching area per LB
  - Even with p>0.5

Relation?

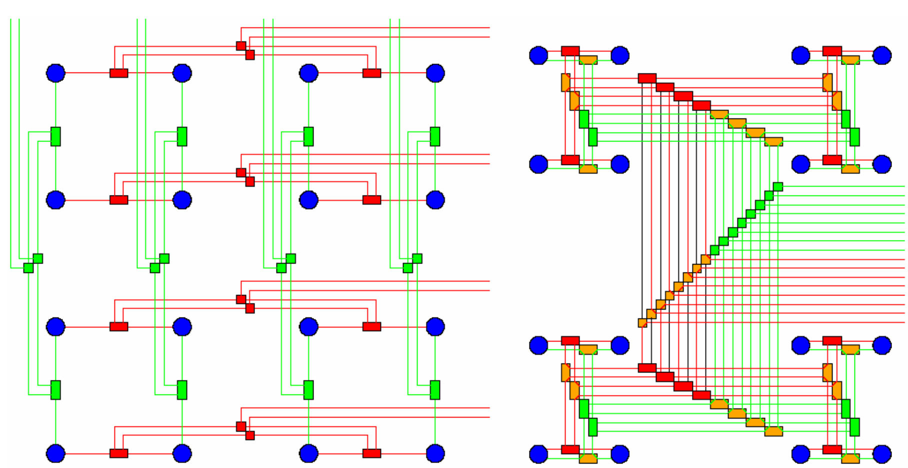
How Related?

- What lessons translate amongst networks?
- Once understand design space
  - Get closer together
- Ideally
  - One big network design we can parameterize

MoT → HSRA (P=0.5)

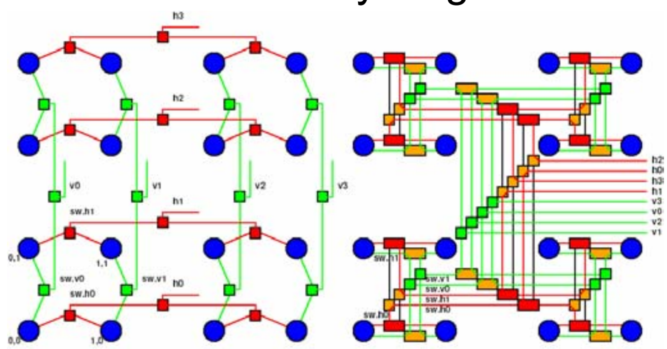


MoT→HSRA (p=0.75)

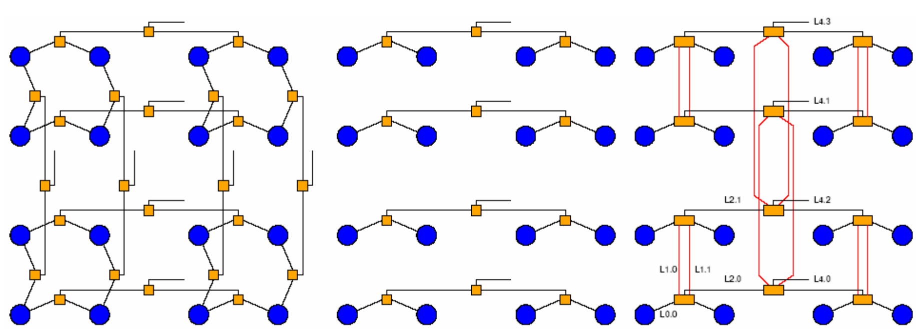


MoT → HSRA

- A C MoT maps directly onto a 2C HSRA
  - Same p's
- HSRA can route anything MoT can

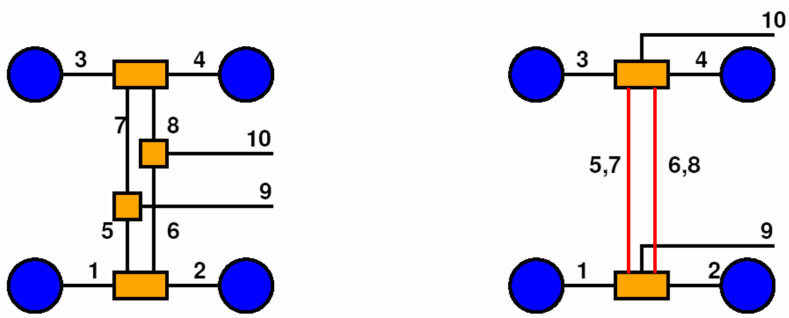


HSRA → MoT

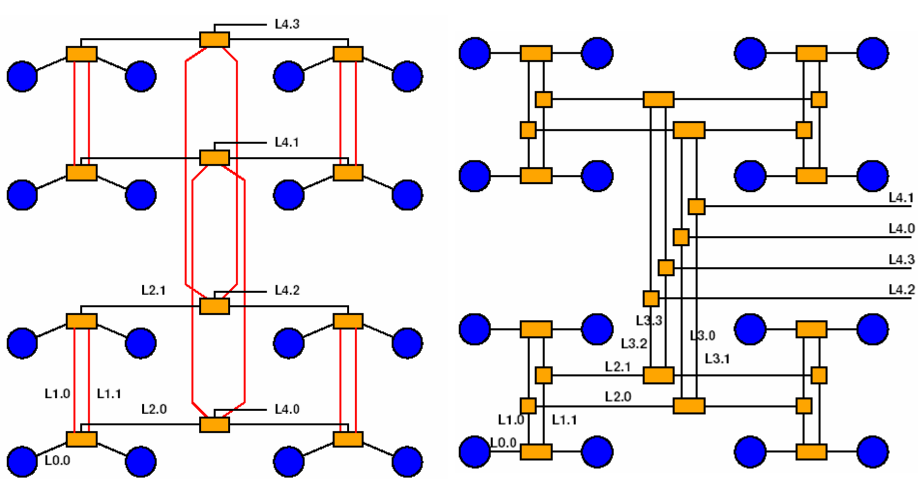


- Decompose and look at rows
- Add homogeneous, upper-level corner turns

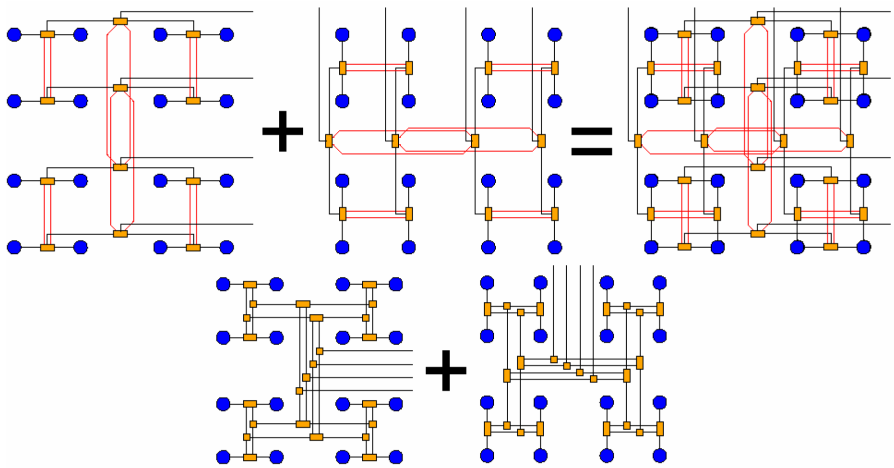
HSRA→MoT



HSRA→MoT

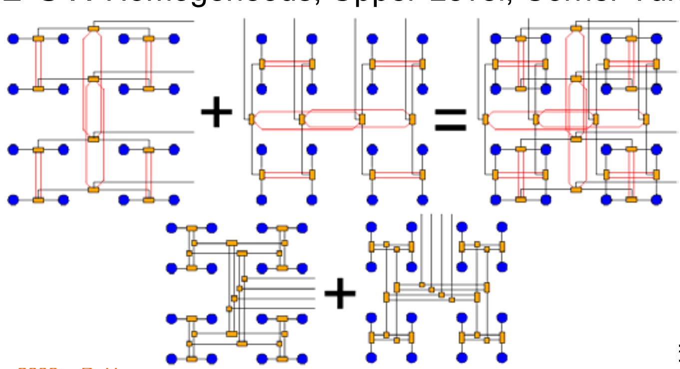


HSRA→MoT

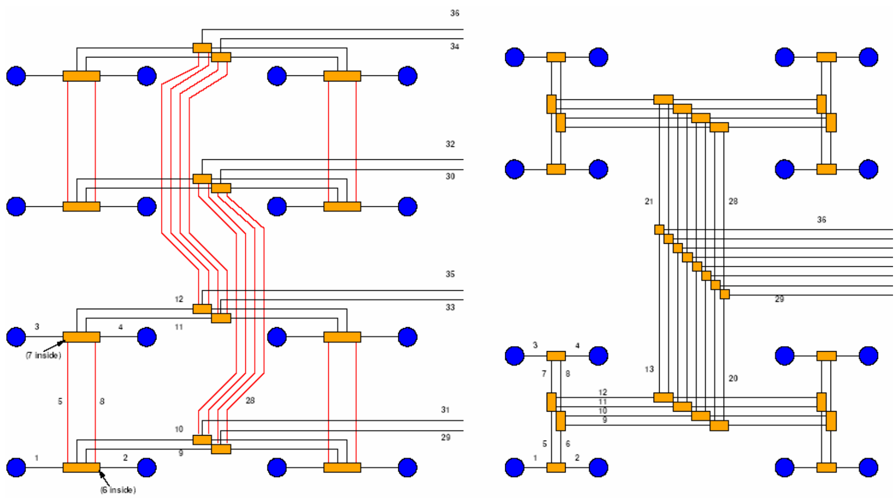


HSRA→MoT

- HSRA + $HSRA^T$ = MoT w/ H-UL-CT
  - Same C, P
  - H-UL-CT: Homogeneous, Upper-Level, Corner Turns

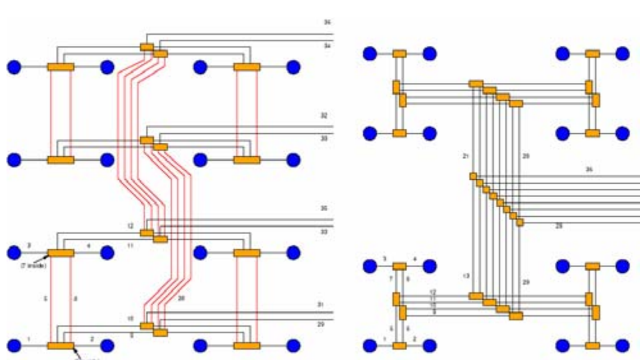


HSRA→MoT (p=0.75)



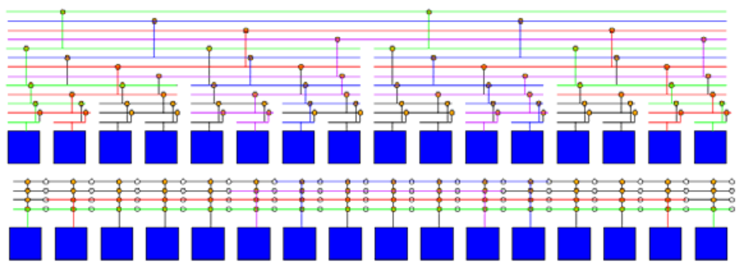
HSRA→MoT (p=0.75)

- Can organize HSRA as MoT
- P>0.5 MoT layout
  - Tells us how to layout p>0.5 HSRA



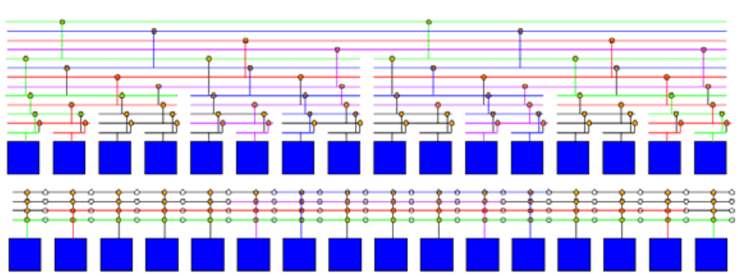
MoT vs. Mesh

- MoT has Geometric Segment Lengths
- Mesh has flat connections
- MoT must climb tree
  - Parameterize w/ flattening
- MoT has $O(N^{p-0.5})$ less switches



MoT vs. Mesh

- Wires
  - Asymptotically the same (p>0.5)
  - Cases where Mesh requires constant less
  - Cases where require same number



Admin

- Monday = President's Day Holiday
  - No Class
  - (CS Systems down for Maintenance)
  - Assignment due Wed. as a result

Big Ideas

- Networks driven by same wiring requirements
  - Have similar wiring asymptotes
- Can bound
  - Network differences
  - Worst-case mesh global routing
- Hierarchy structure allows to save switches
  - $O(N)$ vs. $\Omega(N^{p+0.5})$