

# CS137: Electronic Design Automation

Day 9: February 9, 2004  
Partitioning  
(Intro, KLFM)



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## Today

- Partitioning
  - why important
  - practical attack
  - variations and issues

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## Motivation (1)

- Divide-and-conquer
  - trivial case: decomposition
  - smaller problems easier to solve
    - net win, if super linear
    - $\text{Part}(n) + 2 \times T(n/2) < T(n)$
  - problems with sparse connections or interactions
  - Exploit structure
    - limited cutsize is a common structural property
    - random graphs would **not** have as small cuts

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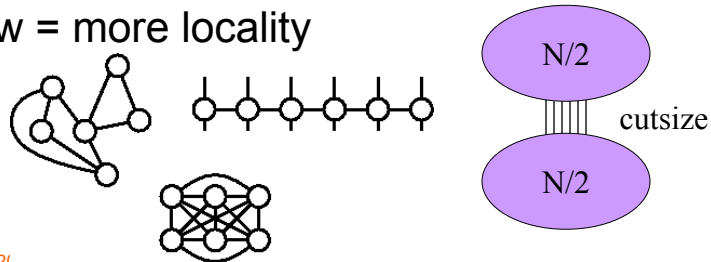
## Motivation (2)

- Cut size (bandwidth) can determine area
- Minimizing cuts
  - minimize interconnect requirements
  - increases signal locality
- Chip (board) partitioning
  - minimize IO
- Direct basis for placement

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# Bisection Bandwidth

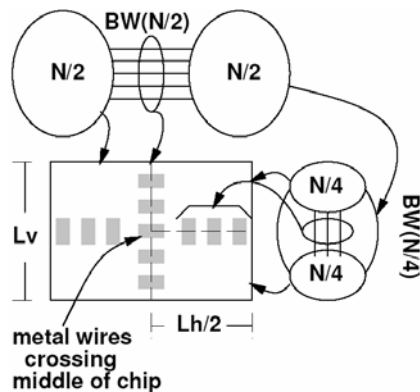
- Partition design into two equal size halves
- Minimize wires (nets) with ends in both halves
- Number of wires crossing is **bisection bandwidth**
- lower bw = more locality



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# Interconnect Area

- Bisection is lower-bound on IC width
  - Apply wire dominated
- (recursively)



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## Classic Partitioning Problem

- **Given:** netlist of interconnect cells
- Partition into two (roughly) equal halves (A,B)
- minimize the number of nets shared by halves
- “Roughly Equal”
  - balance condition:  $(0.5-\delta)N \leq |A| \leq (0.5+\delta)N$

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## Balanced Partitioning

- NP-complete for general graphs
  - [ND17: Minimum Cut into Bounded Sets, Garey and Johnson]
  - Reduce SIMPLE MAX CUT
  - Reduce MAXIMUM 2-SAT to SMC
  - Unbalanced partitioning poly time
- Many heuristics/attacks

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## KL FM Partitioning Heuristic

- Greedy, iterative
  - pick cell that decreases cut and move it
  - repeat
- small amount of non-greediness:
  - look past moves that make locally worse
  - randomization

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## Fiduccia-Mattheyses (Kernighan-Lin refinement)

- Start with two halves (random split?)
- Repeat until no updates
  - Start with all cells free
  - Repeat until no cells free
    - Move cell with largest gain (**balance allows**)
    - Update costs of neighbors
    - Lock cell in place (record current cost)
  - Pick least cost point in previous sequence and use as next starting position
- Repeat for different random starting points

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# Efficiency

Tricks to make efficient:

- Pick move candidate with little work
- Update costs on move cheaply
- Efficient data structure
  - update costs cheap
  - cheap to find next move

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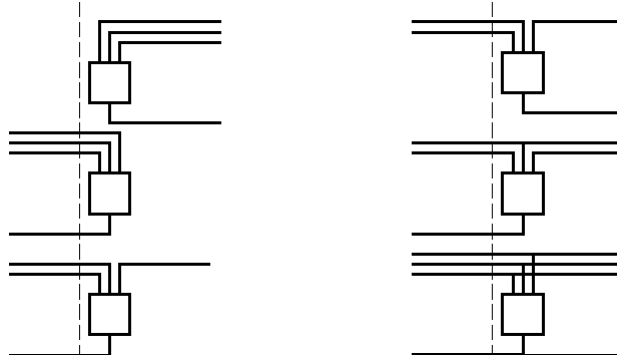
# Ordering and Cheap Update

- Keep track of Net gain on node == delta net crossings to move a node
  - cut cost after move = cost - gain
- Calculate node gain as  $\Sigma$  net gains for all nets at that node
  
- Sort by gain

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# FM Cell Gains

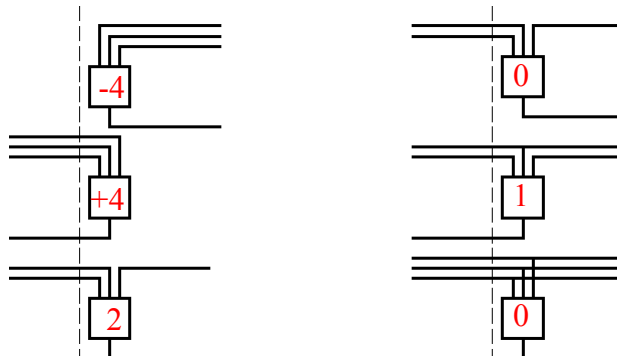
Gain = Delta in number of nets crossing between partitions



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# FM Cell Gains

Gain = Delta in number of nets crossing between partitions



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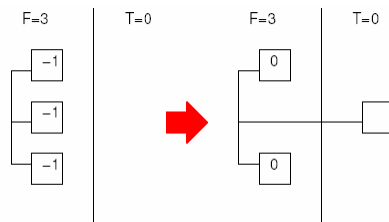
## After move node?

- Update cost each
  - $\text{Newcost} = \text{cost} - \text{gain}$
- Also need to update gains
  - on all nets attached to moved node
  - but moves are nodes, so push to
    - all nodes affected by those nets

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## FM Recompute Cell Gain

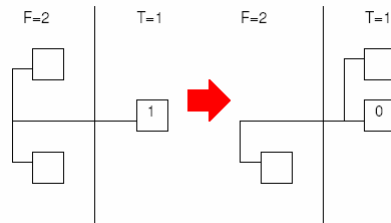
- For each net, keep track of number of cells in each partition [F(net), T(net)]
- Move update:(for each net on moved cell)
  - if  $T(\text{net}) = 0$ , increment gain on F side of net
  - (think  $-1 \Rightarrow 0$ )



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## FM Recompute Cell Gain

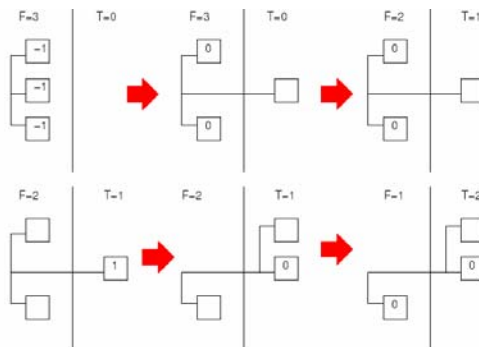
- For each net, keep track of number of cells in each partition [F(net), T(net)]
- Move update:(for each net on moved cell)
  - if T(net)==0, increment gain on F side of net
    - (think -1 => 0)
  - if T(net)==1, decrement gain on T side of net
    - (think 1=>0)



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## FM Recompute Cell Gain

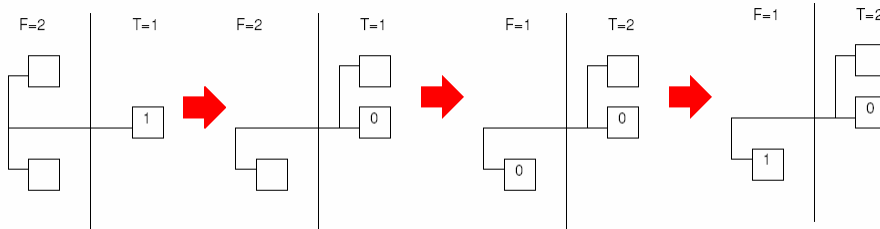
- Move update:(for each net on moved cell)
  - if T(net)==0, increment gain on F side of net
  - if T(net)==1, decrement gain on T side of net
  - decrement F(net), increment T(net)



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## FM Recompute Cell Gain

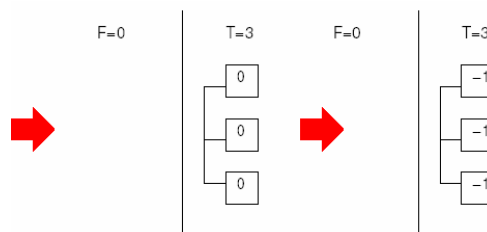
- Move update:(for each net on moved cell)
  - if  $T(\text{net})=0$ , increment gain on F side of net
  - if  $T(\text{net})=1$ , decrement gain on T side of net
  - decrement  $F(\text{net})$ , increment  $T(\text{net})$
  - if  $F(\text{net})=1$ , increment gain on F cell



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## FM Recompute Cell Gain

- Move update:(for each net on moved cell)
  - if  $T(\text{net})=0$ , increment gain on F side of net
  - if  $T(\text{net})=1$ , decrement gain on T side of net
  - decrement  $F(\text{net})$ , increment  $T(\text{net})$
  - if  $F(\text{net})=1$ , increment gain on F cell
  - if  $F(\text{net})=0$ , decrement gain on all cells (T)



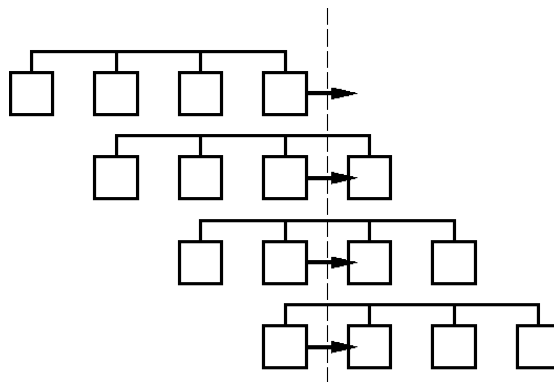
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# FM Recompute Cell Gain

- For each net, keep track of number of cells in each partition [F(net), T(net)]
- Move update:(for each net on moved cell)
  - if T(net)==0, increment gain on F side of net
    - (think -1 => 0)
  - if T(net)==1, decrement gain on T side of net
    - (think 1=>0)
  - decrement F(net), increment T(net)
  - if F(net)==1, increment gain on F cell
  - if F(net)==0, decrement gain on all cells (T)

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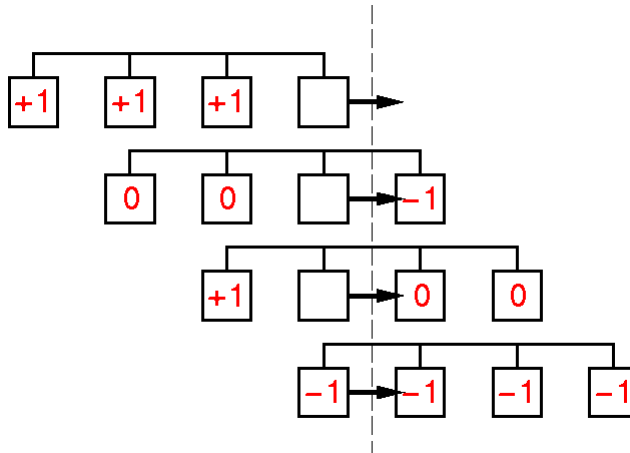
# FM Recompute (example)



(work through before advancing to next slide)

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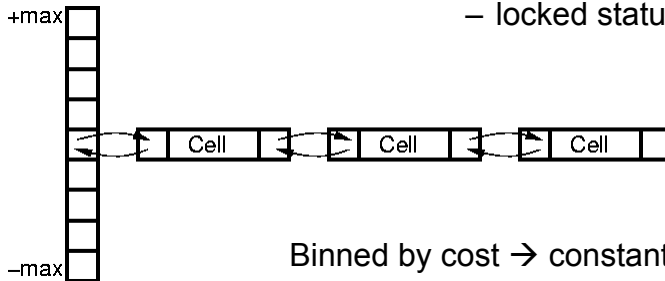
# FM Recompute (example)



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# FM Data Structures

- Partition Counts A,B
- Two gain arrays
  - One per partition
  - **Key:** constant time cell update
- Cells
  - successors (consumers)
  - inputs
  - locked status



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# FM Optimization Sequence

(ex)

+3	+2	-1
+3	+1	-1
+2	0	-1
+2	-1	0
+1	0	+1
0	-1	-1
+1	-1	-1
0	-2	-2
0	-2	-2
-1	-1	-2
+1	0	-1
-1	-1	-2
-2	-2	-1
-2	-3	-3
-3	-3	-3
-3	-3	-3
<b>+12</b>	<b>+3</b>	<b>0</b>

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## FM Running Time?

- Randomly partition into two halves
- Repeat until no updates
  - Start with all cells free
  - Repeat until no cells free
    - Move cell with largest gain
    - Update costs of neighbors
    - Lock cell in place (record current cost)
  - Pick least cost point in previous sequence and use as next starting position
- Repeat for different random starting points

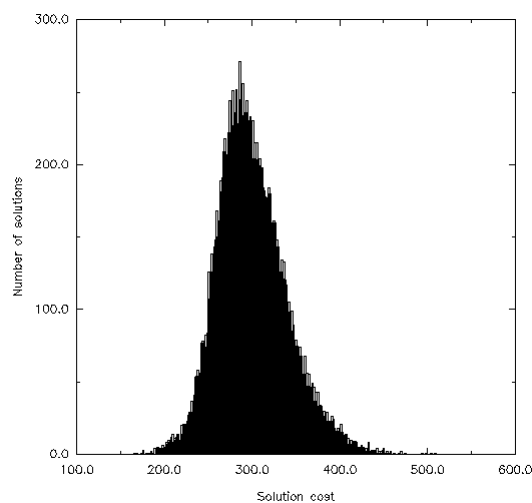
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# FM Running Time

- **Claim:** small number of passes (constant?) to converge
- Small (constant?) number of random starts
- N cell updates each round (swap)
- Updates K + fanout work (avg. fanout K)
  - assume K-LUTs
- Maintain ordered list  $O(1)$  per move
  - every io move up/down by 1
- Running time:  $O(KN)$ 
  - Algorithm significant for its speed (more than quality)

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# FM Starts?



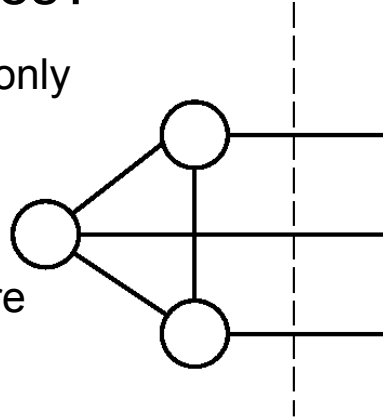
So, FM gives  
a not bad  
solution  
quickly

21K random starts, 3K network -- Alpert/Kahng

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## Weaknesses?

- Local, incremental moves only
  - hard to move clusters
  - no lookahead
  - [see example]
- Looks only at local structure



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## Improving FM

- Clustering
- Technology mapping
- Initial partitions
- Runs
- Partition size freedom
- Replication

Following comparisons from Hauck and Boriello '96

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# Clustering

- Group together several leaf cells into cluster
- Run partition on clusters
- Uncluster (keep partitions)
  - iteratively
- Run partition again
  - using prior result as starting point
    - instead of random start

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# Clustering Benefits

- Catch local connectivity which FM might miss
  - moving one element at a time, hard to see move whole connected groups across partition
- Faster (smaller N)
  - METIS -- fastest research partitioner exploits heavily
  - FM work better w/ larger nodes (???)

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## How Cluster?

- Random
  - cheap, some benefits for speed
- Greedy “connectivity”
  - examine in random order
  - cluster to most highly connected
  - 30% better cut, 16% faster than random
- Spectral (next time)
  - look for clusters in placement
  - (ratio-cut like)
- Brute-force connectivity (can be  $O(N^2)$ )

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## LUT Mapped?

- Better to partition before LUT mapping.



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## Initial Partitions?

- Random
- Pick Random node for one side
  - start imbalanced
  - run FM from there
- Pick random node and Breadth-first search to fill one half
- Pick random node and Depth-first search to fill half
- Start with Spectral partition

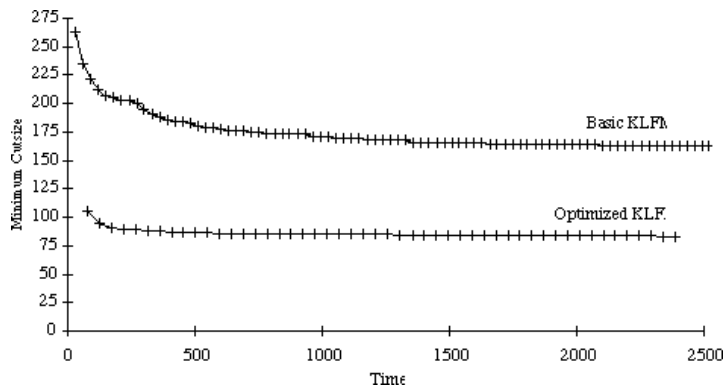
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## Initial Partitions

- If run several times
  - pure random tends to win out
  - more freedom / variety of starts
  - more variation from run to run
  - others trapped in local minima

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# Number of Runs



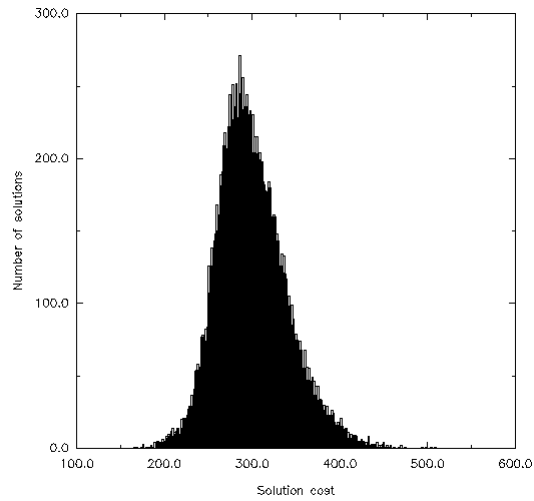
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# Number of Runs

- 2 - 10%
- 10 - 18%
- 20 < 20% (2% better than 10)
- 50 (4% better than 10)
- ...but?

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# FM Starts?

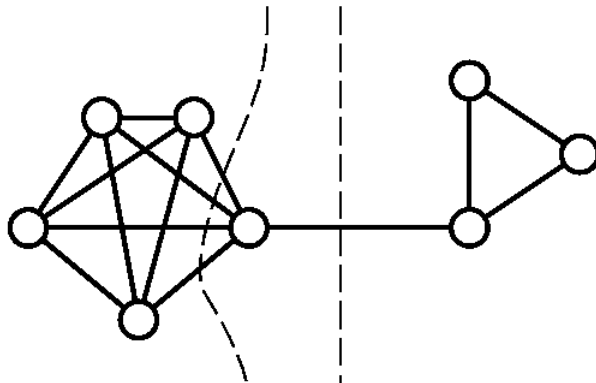


21K random starts, 3K network -- Alpert/Kahng

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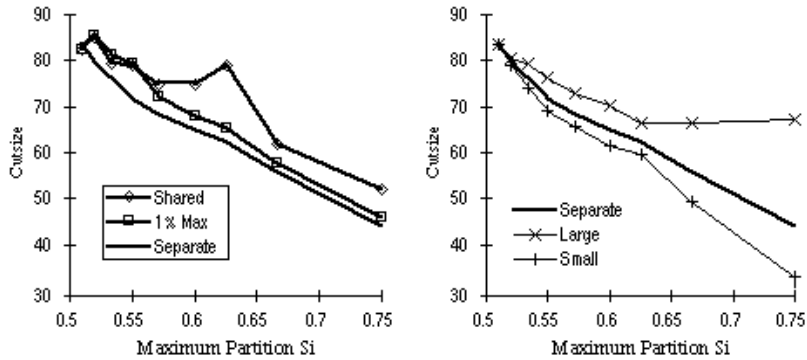
# Unbalanced Cuts

- Increasing slack in partitions
  - may allow lower cut size



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# Unbalanced Partitions

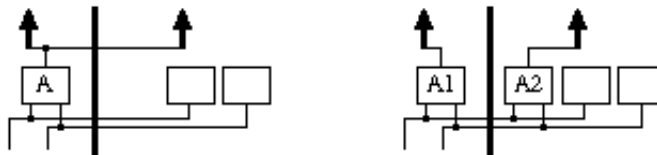


Following comparisons from Hauck and Boriello '96

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# Replication

- Trade some additional logic area for smaller cut size
  - Net win if wire dominated

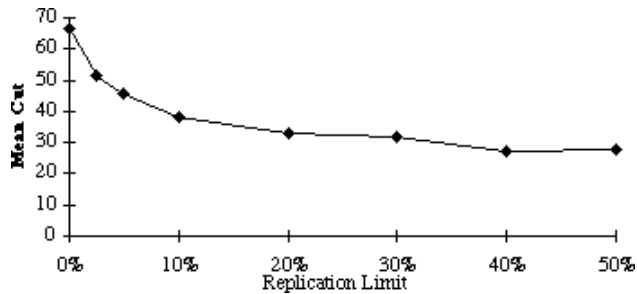


Replication data from: Enos, Hauck, Sarrafzadeh '97

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# Replication

- 5% → 38% cut size reduction
- 50% → 50+% cut size reduction



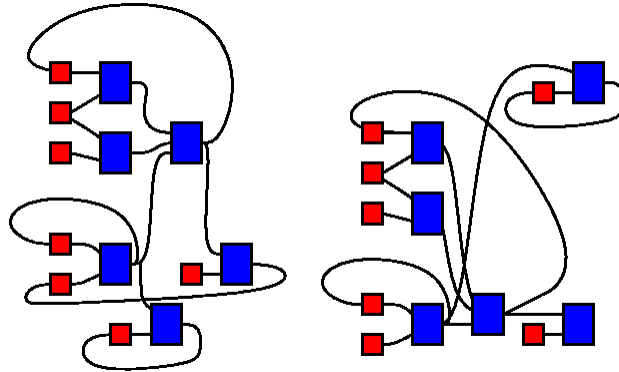
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## What Bisection doesn't tell us

- Bisection bandwidth purely geometrical
- No constraint for delay
  - *i.e.* a partition may leave critical path weaving between halves

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# Critical Path and Bisection



Minimum cut may cross critical path multiple times.  
Minimizing long wires in critical path => increase cut size.

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## So...

- Minimizing bisection
  - good for area
  - oblivious to delay/critical path

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# Partitioning Summary

- Decompose problem
- Find locality
- NP-complete problem
- linear heuristic (KLFM)
- many ways to tweak
  - Hauck/Boriello, Karypis
- even better with replication
- only address cut size, not critical path delay

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# Admin

- Assignment #3, #4 out today
  - Based on 2-level optimization
  - ...but stuff covering now may be highly relevant to your solutions
    - *E.g.* is group formation a partitioning problem?
- Today's supplemental references linked to reading
  - (Hauck, Karypis, Alpert...)

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## Today's Big Ideas:

- Divide-and-Conquer
- Exploit Structure
  - Look for sparsity/locality of interaction
- Techniques:
  - greedy
  - incremental improvement
  - randomness avoid bad cases, local minima
  - incremental cost updates (time cost)
  - efficient data structures