

# CS137: Electronic Design Automation

Day 2: March 31, 2004  
Dual Objective  
Dynamic Programming



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## Today

- Cover and Place
  - Linear
    - GAMA
    - Optimal Tree-based
  - Area and Time
    - covering for
    - ...and linear placement
  - Two Dimensional
    - Lily

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# Covering Review

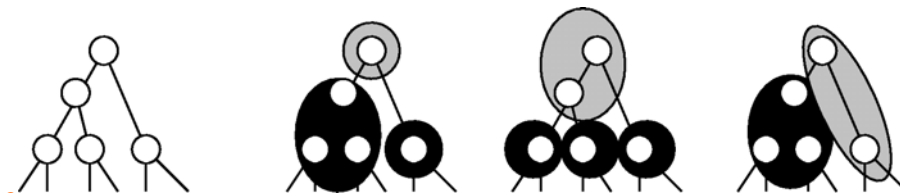
- Use dynamic programming to optimally cover trees
  - problem decomposable into subproblems
  - optimal solution to each are part of optimal
  - no interaction between subproblems
  - small number of distinct subproblems
  - **single optimal solution to subproblem**
- Break DAG into trees then cover optimally

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# Covering Basics

Basic Idea:

- Assume have optimal solution to all subproblems smaller than current problem
- try all ways of implementing current root
  - each candidate solution is new gate + previously solve subtrees
  - pick best (smallest area, least delay, least power)



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# Placement

- How do we integrate placement into this covering process?

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## GaMa - Linear Placement

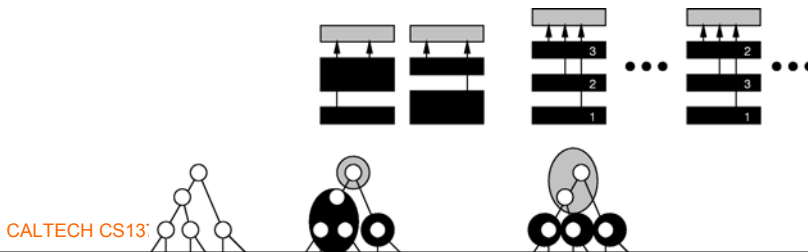
- **Problem:** cover and place datapaths in rows of FPGA-like cells to minimize area, delay
- Datapath width extends along one dimension (rows)
- Composition is 1D along other dimension (columns)
- Always covering SIMD row at a time

[Callahan/FPGA'98]

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## Basic Strategy

- Restrict each subtree to a contiguous set of rows
- Build up placement for subtree during cover
- When consider cover, also consider all sets of arrangements of subtrees
  - effectively expands library set



## Simultaneous Placement Benefits

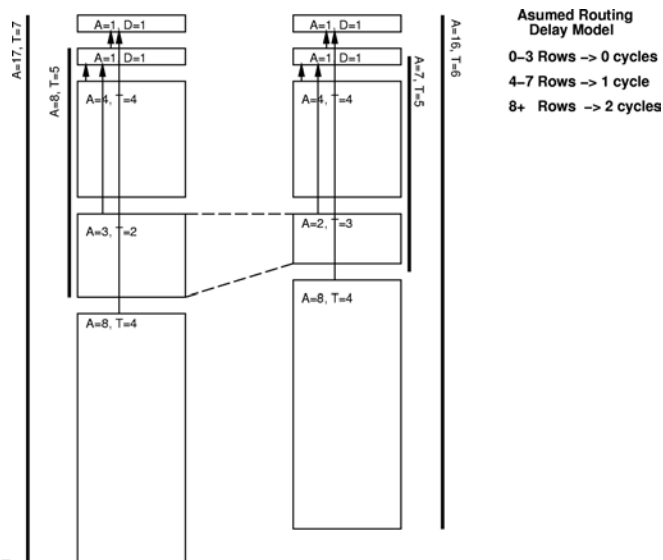
- Know real delay (including routing) during covering
  - make sure critical logic uses fastest inputs
  - ...shortest paths
- Know adjacency
  - can use special resources requiring adjacent blocks

# GaMa Properties

- Operates in time linear in graph size
  - $O(|\text{rule set}| * |\text{graph nodes}|)$
- Finds area-optimum for restricted problem
  - trees with contiguous subtrees
- As is, may not find delay optimum

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# GaMa Delay Example



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## GaMa Delay Problem

- Area can affect delay
- Doesn't know when to pick worse delay to reduce area
  - make non-critical path subtree slower/smaller
  - so overall critical path will be close later
- Only tracking single objective
- **Fixable as next technique demonstrates**

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## GaMa Results

- Comparable result quality (area, time) to running through Xilinx tools
- Placement done in **seconds** as opposed to minutes to hours for Xilinx
  - simulated annealing, etc.
  - not exploiting datapath regularity

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# Simultaneous Mapping and Linear Placement of Trees

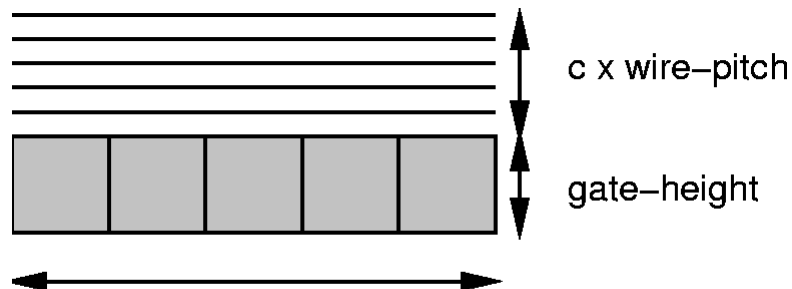
- **Problem:** cover and place standard cell row minimizing area
- Area: cell width and cut width
- Technique: combine DP-covering with DP-tree layout

[Lou+Salek+Pedram/ICCAD'97]

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## Task

- Minimize:
  - Area = gate-width \* (gate-height + c \* wire-pitch)

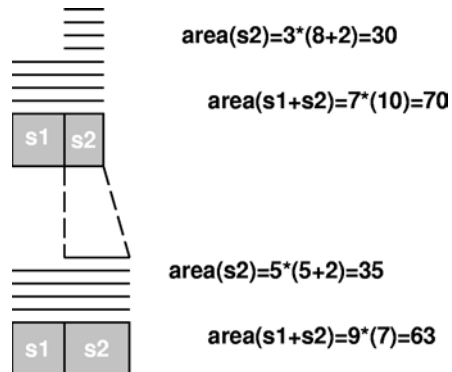


CALTECH C

gate-wdth

## Composition Challenge

- Minimum area solution to subproblems does **not** necessarily lead to minimum area solution:



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## Minimize Area

- Two components of area:
  - gate-area
  - cut-width
- Unclear during mapping when need
  - a smaller gate-area
  - vs. a smaller cut-width
    - at the expense of (local) cell area
  - (same problem as area vs. delay in GaMa)

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# Strategy

- Recognize that these are incomparable objectives
  - neither is strictly superior to other
  - keep all solutions
  - discard only inferior (dominated) solutions

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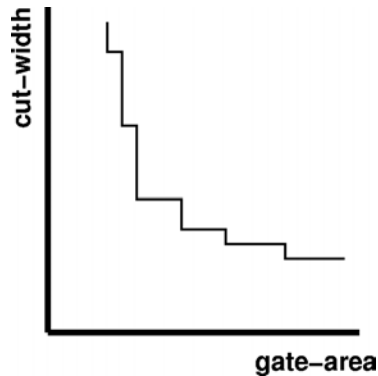
# Dominating/Inferior Solutions

- A solution is **dominated** if there is another solution strictly superior in all objectives
  - $A=3, T=2$     $A=2, T=3$ 
    - neither dominates
  - $A=3, T=3$     $A=3, T=2$     $A=2, T=3$ 
    - $A=3, T=3$  is inferior, being dominated by either of the other two solutions

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## Non-Inferior Curve

- Set of dominators defines a curve



This is a recurring theme---often prune work using dominator curve

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## Strategy

- Keep curve of non-inferior area-cut points
- During DP
  - build a new curve for each subtree
  - by looking at solution set intersections
    - cross product set of solutions from each subtrees feeding into this subtree

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## Consequences

- More work per graph point
  - keeping and intersecting many points
- Theory: points<sup>(fanin)</sup> \* gates
- Points  $\leq$  range of solutions in smallest dimension
- e.g. points  $\leq$  number of different cut-widths

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## Algorithm: Tree Cover+Place

- For each tree node from leafs
  - For each gate cover
    - For each non-inferior point in fanin-subtrees
      - compute optimal tree layout
      - keep non-inferior points (cutwidth, gate-area)
- Optimal Tree Layout
  - Yannakakis/JACM v32n4p950, Oct. 1985

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## Time Notes

- Computing Optimal Tree layout:  
 $O(N \log(N))$
- Per node:  $O(\text{cutwidth}^{(\text{fanin})} * N * \log(N))$
- Loose bound
  - possible to tighten?
  - less points and smaller “N” in tree for earlier subproblems
  - higher fanin → less depth → more use of small “N” for linear layout problems

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## Empirical Results

- Claim: 20% area improvement

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## Covering for Area and Delay

- Previously saw was hard to do DP to
  - simultaneously optimize for area and delay
  - properly generate area-time tradeoffs
- **Problem:**
  - whether or not needed a fast path
  - not clear until saw speed of siblings

[Chaudhary+Pedram/DAC'92]

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## Strategy

- Use same technique as just detailed for
  - gate-area + cutwidth
- *i.e.* -- at each tree cover
  - keep all non-inferior points
    - (effectively the full area-time curve)
  - as cover, intersect area-time curves to generate new area-time curve
- When get to a node
  - can pick smallest implementation for a child node that does not increase critical path

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## Points to Keep

- Usually small variance in times
  - if use discrete model like LUT delays, only a small number of different times
  - if use continuous model, can get close to optimum by discretizing and keeping a fixed set
- Similarly, small total variance in area
  - e.g. factor of 2-3
  - discretizing, gets close w/out giving up much
- Discretized: run in time linear in N
  - assuming bounded fanin gates

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## GaMa -- Optimal Delay

- Use this technique in GaMa
  - solve delay problem
  - get good area-delay tradeoffs
  - GARP has a discrete timing model
    - so already have small spread
  - for conventional FPGA
    - will have to discretize

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# Covering and Linear Placement for Area and Delay

- Have both
  - cut-width + gate-area affects
  - delay tradeoff
- Result
  - have three objectives to minimize
    - cut-width
    - gate-area
    - gate-delay

[Lou+Salek+Pedram/ICCAD'97]

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## Strategy

- Repeat trick:
  - keep non-inferior points in three-space
    - <cut-width,gate-area,delay>
  - Intersect spaces to compute new cover spaces
  - May really need to discretize points to limit work

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## Note

- Delay calculation:
  - assumes delay in gates and fanout
  - fanout effect makes heuristic
    - maybe iterate/relax?
  - **ignores distance**
- “Optimal” tree layout algorithm being used
  - is optimal with respect to cut-width
  - **not optimal** with respect to critical path wire length

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## Empirical Results

- Mapping for delay:
  - 20% delay improvement
  - achieving effectively same area
    - (of alternative, not of self targeting area)

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## Two Dimensions?

- Both so far, one-dimensional
- One-dimensional
  - nice layout restrictions
  - simple metric for delay
  - simple metric for area
- How extend to two dimensions?

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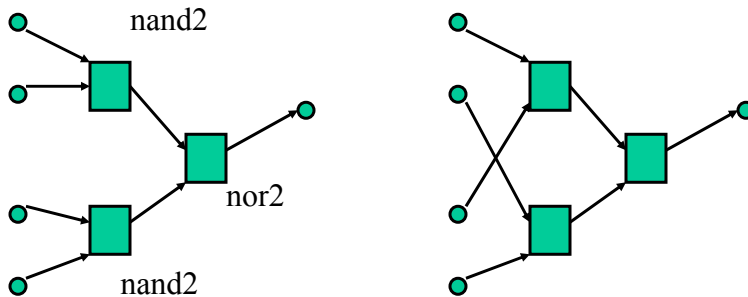
## 2D Cover and Place

- **Problem:** cover and place in 2D to minimize area (delay)
- Area: gate area + “wirelength” area
- Delay: gate delay + estimated wire delay

[Pedram+Bhat/DAC'91]  
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## Example

- Covering wrt placement matters



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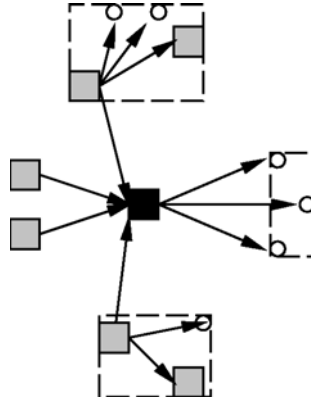
## Strategy

- Relax placement during covering
- Initially place unmapped using constructive placement (CS137a)
- Cover via dynamic programming
- When cover a node,
  - fanins already visited
  - calculate new placement
    - Center of Mass...like Force Directed
- Periodically re-calculate placement
- Use estimated/refined placements to get area, delay

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# Incremental Placement

- Place newly covered nodes so as to minimize wire lengths (critical path delay?)



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# Empirical Results

- In 1 $\mu$ m
  - 5% area reduction
  - 8% delay reduction
- Not that inspiring
  - ...but this was in the **micron** era
  - probably have a bigger effect today

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# Summary

- Can consider placement effects while covering
- Many problems can't find optimum by minimizing single objective
  - delay (area effects)
  - area (cutwidth effects)
- Can adapt DP to solve
  - keep all non-inferior points
  - can keep polynomial time
    - if very careful, primarily increase constants

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# Admin

- Project Selection due Friday
  - Email short note
- Concept Generation lecture on Monday
  - Handed out reading last time

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## Big Ideas:

- Simultaneous optimization
- Multi-dimensional objectives
  - dominating points (inferior points)
  - use with dynamic programming
- Exploit stylized problems can solve optimally
- Phase Ordering: estimate/iterate