CS137:
Electronic Design Automation

Day 15: March 4, 2002

Two-Level Logic-Synthesis

Today

- Two-Level Logic Optimization
  - Problem
  - Definitions
  - Basic Algorithm: Quine-McClusky
  - Improvements

Problem

- **Given**: Expression in combinational logic
- **Find**: Minimum (cost) sum-of-products expression
- Ex.
  - Y=a*b*c + a*b*/c + a*/b*c
  - Y=a*b + a*c

EDA Use

- Minimum size PAL, PLA, …
- Minimum number of gates for two-level implementation
- Starting point for multi-level optimization

Complexity

- Set covering problem
  - NP-hard

Cost

- PLA/PAL - first order
  - number of product terms
- Abstract (mis, sis)
  - number of literals
    - cost(y=a*b+a*/c)=4
- General (simple, multi-level)
  - ∑cost(product-term)
    - e.g. nand2=4, nand3=5,nand4=6…

Terminology (1)

- Literals -- a, /a, b, /b, ….
  - Qualified, single inputs
- Minterms --
  - full set of literals covering one input case
  - in y=a*b+a*c
    - a*b*c
    - a*/b*c

Terminology (2)

- Cube:
  - product covering one or more minterms
  - Y=a*b+a*c
  - cubes:
    - a*b*c abc
    - a*b ab
    - a*c ac

Terminology (3)

- Cover:
  - set of cubes
  - sum products
  - {abc, a/bc, ab/c}
  - {ab,ac}

Truth Table

- Also represent function

a	b	c	y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Specify on-set only

a	b	c	y
1	0	1	1
1	1	0	1
1	1	1	1

Cube/Logic Specification

- Canonical order for variables
- Use {0,1,-} to indicate input appearance in cube
  - 0 - inverted
  - 1 - not inverted
  - - - not present

abc 111
a/bc 101
ac 1-1

```
a b c   y
1 0 1   1
1 1 0   1
1 1 1   1
```

```
1 0 1   1
1 1 0   1
1 1 1   1
```

```
1 - 1   1
1 1 -   1
```

```
1 - 1
1 1 -
```


In General

- Three sets:
  - on-set (must be set to one by cover)
  - off-set (must be set to zero by cover)
  - don't care set (can be zero or one)
- Don't Cares
  - allow freedom in covering (reduce cost)
  - arise from cases where value doesn't matter
    - *e.g.* outputs in non-existent FSM state
    - data bus value when not driving bus

Multiple Outputs

Truth Table:

a b	y x
0 0	1 1
0 1	0 0
1 0	0 0
1 1	0 1

Convert to single-output problem

a b	y x	o
0 0	1 -	1
0 0	- 1	1
0 1	0 -	1
0 1	- 0	1
1 0	0 -	1
1 0	- 0	1
1 1	0 -	1
1 1	- 1	1

On-set for result

001-
00-1
010 -
01- 0
100 -
10- 0
110 -
11- 1

Multiple Outputs

- Can reduce to single output case
  - write equations on inputs and each output
    - with onset for relation being true
  - after cover
    - remove literals associated with outputs

Multiple Outputs

- Could Optimize separately
- By optimizing together
  - Maximize sharing of cubes/product-terms

Multiple Outputs

- Consider:
  - X=/a/b+ab+ac
  - Y=/bc
- Also:
  - X=/a/b+ab+/bc

000
001
101
110
111

Smaller PLA w/ shared product term

001 /a/b /bc
000 /a/b
101 /bc
110 ab
111 ab

Prime Implicants

- Implicant -- cube in on-set
  - (not entirely in don't-case set)
- Prime Implicant -- implicant, not contained in any other cube
  - for y=a*b+a*c
    - a*b is a prime implicant
    - a*b*c is not a prime implicant (contained in ab, ac)
  - *I.e.* largest cube still in on-set (on+dc-sets)

Prime Implicants

- Minimum cover will be made up of primes
  - less products if cover more
  - less literals in prime than contained cubes
- Necessary but not sufficient that minimum cover contain only primes
  - y=ab+ac+b/c
  - y=ac+b/c
- Number of PI's can be exponential in input size
  - more than minterms, even!
  - Not all PI's will be in optimum cover

Restate Goal

- Goal in terms of PIs
  - Find minimum size set of PIs which cover the on-set.

Essential Prime Implicants

- Prime Implicant which contains a minterm not covered by any other PI
  - Essential PI **must** occur in any cover
  - y=ab+ac+b/c
  - ab 11- 110 111
  - ac 1-1 101 111 * essential (only 101)
  - b/c -10 110 010 * essential (only 010)

Computing Primes

- Start with minterms
  - for on-set and dc-set
- merge pairs (distance one apart)
- for each pair merged,
  - mark source cubes as covered
- repeat merging for resulting cube set
  - until no more merging possible
- retain all unmarked cubes which aren't entirely in dc-set

Compute Prime Example

0 0000
5 0101
7 0111
8 1000
9 1001
10 1010
11 1011
14 1110
15 1111

Compute Prime Example

0 0000	0, 8 -000
5 0101	5, 7 01-1
7 0111	7,15 -111
8 1000	8, 9 100-
9 1001	8,10 10-0
10 1010	9,11 10-1
11 1011	10,11 101-
14 1110	10,14 1-10
15 1111	11,15 1-11
	14,15 111-

Compute Prime Example

0 0000	0, 8 -000	0, 8 -000			/b/c/d
5 0101	5, 7 01-1	5, 7 01-1			/abd
7 0111	7,15 -111	7,15 -111			bcd
8 1000	8, 9 100-	8, 9 100-	*	8, 9,10,11 10--	a/b
9 1001	8,10 10-0	8,10 10-0	*		ac
10 1010	9,11 10-1	9,11 10-1	*		
11 1011	10,11 101-	10,11 101-	*	10,11,14,15 1-1-	
14 1110	10,14 1-10	10,14 1-10	*		
15 1111	11,15 1-11	11,15 1-11	*		
	14,15 111-	14,15 111-	*		

Covering Matrix

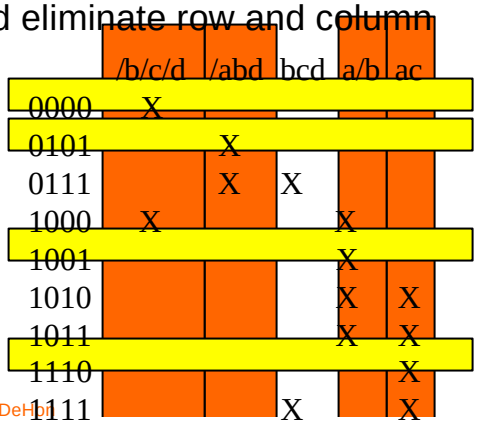
- Minterms × Prime Implicants

Goal: minimum cover

	/b/c/d	/abd	bcd	a/b	ac
0000	X				
0101		X			
0111		X	X		
1000	X			X	
1001				X	
1010				X	X
1011				X	X
1110					X
1111			X		X

Essential Reduction

- Must pick essential PI
  - pick and eliminate row and column



Essential Reduction

- This case:
  - Cover determined by essentials
- General case:
  - Reduces size of problem
  - These are easy…

Dominators: Column

- If a column (PI) covers the same or strictly more than another column
  - can remove **dominated** column

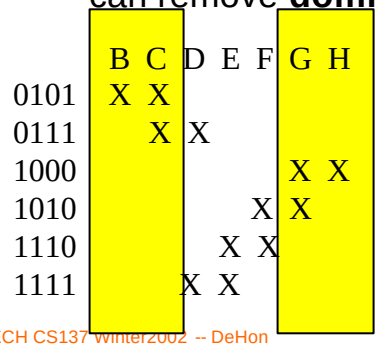
	B	C	D	E	F	G	H
0101	X	X					
0111		X	X				
1000						X	X
1010					X	X	
1110				X	X		
1111			X	X			

C dominates B

G dominates H

Dominators: Column

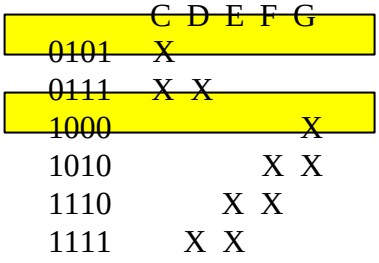
- If a column (PI) covers the same or strictly more than another column
  - can remove **dominated** column



	C	D	E	F	G
0101	X				
0111	X	X			
1000					X
1010				X	X
1110			X	X	
1111		X	X		

New Essentials

- Dominance reduction may yield new Essential PIs



C,G now essential

	C	D	E	F	G
1110			X	X	
1111		X	X		

E dominates D and F

Cover = {C,E,G}

Dominators: Row

- If a row has the same (or strictly more) PIs than another row, the larger row dominantes
  - we can remove the **dominating** row
    - (NOTE OPPOSITE OF COLUMN CASE)

	C	D	E	F	G
0101	X				
0111	X	X			
1000					X
1010				X	X
1110			X	X	
1111			X	X	

0111 dominates 0101
remove 0111

1010 dominates 1000
remove 1010

Cyclic Core

- After applying reductions
  - essential
  - column dominators
  - row dominators
- May still have a non-trivial covering matrix
- How do we move forward from here?

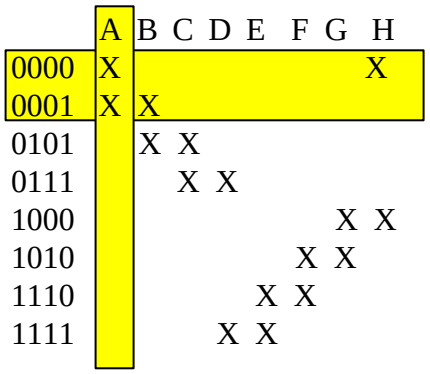
Example

	A	B	C	D	E	F	G	H
0000	X							X
0001	X	X						
0101		X	X					
0111			X	X				
1000							X	X
1010						X	X	
1110					X	X		
1111				X	X			

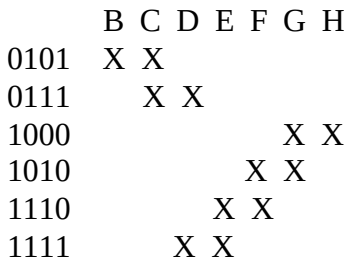
Cyclic Core

- Cannot select (e.g. essential) or exclude (e.g. dominated) a PI definitively.
- Make a guess
  - A in cover
  - A not in cover
- Proceed from there

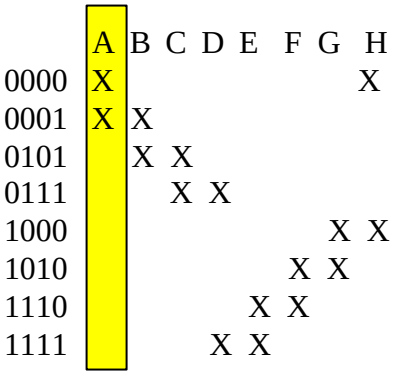
Example



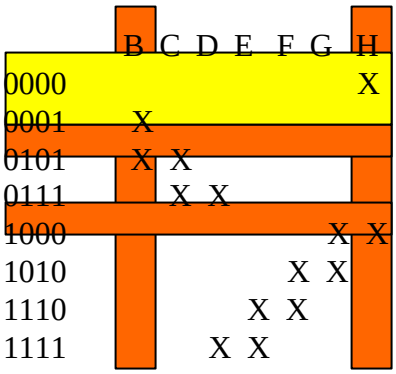
A in Cover:



Example



A not in Cover:



Basic Two-Level Minimization

- Generate Prime Implicants
- Reduce (essential, dominators)
- If not done,
  - pick a cube
  - branch (back to reduce) on selected/not
    - *i.e.* search tree … branch and bound
- Save smallest

Optimization

- Summarize Minterms (signature cubes)
  - rows represent collection of minterms with same primes
- Avoid generating full set of PIs
  - pre-combining dominators during generation
- Branch-and-bound pruning
  - get lower bound on remaining cost of a cover by computing independent set of primes
    - (not necessarily maximal, that would be NP-hard)

Heuristic

- Don't backtrack when select prime for inclusion/exclusion
  - pick cover large set of minterms/signatures
  - weight to select "hard" to cover signatures
- Generate reduced set of PIs
- Iterative improvement

Canonical Form

- Can start with *any* form of logical expression
- Get unique truth-table/minterms
- Problem not sensitive to input statement
  - compare covering (decomposition)
  - compare sequential programming languages
- **Cost:** potentially exponential explosion in minterms/PIs

Summary

- Formulate as covering problem
- Solution space restricted to PIs
- Essentials must be in solution
- Use dominators to further reduce space
- Then branching/pruning to explore rest of PIs
- Ways to reduce work
  - group minterms/PIs together early
  - mostly fall into this general scheme

Big Ideas

- Canonical Form
  - eliminate bias of input specification
- Technique:
  - branch-and-bound
  - dominators
  - use structure of problem to derive reduction between branching selection