CS 101.3: Numerical Geometric Integration

Homework Assignment #5
Due date: Feb 23rd 2009 at the beginning of class.
All code should be submitted by email.

Abstract

In this assignment you will explore the most common multistep methods and derive a few integrators based on them. Please note that the honor code applies: do the derivations yourself. If you have questions email patrickm@cs.caltech.edu - I will be happy to meet upon request.

Adams Method

Derive the Adams-Moulton integrator for $\dot{y} = f(y)$ using $s = 3$ (this should rely on the 3 previous derivatives along with the one implicitly evaluated at the point being solved for). There are several ways to do this, I personally suggest using Mathematica to solve for the coefficients of the polynomial that interpolates the derivatives and then integrating it.

Backwards Difference Method

Derive the integrator for the same system using a 3rd order BDF method (the one in the slides was the 2nd order one). Note that the interpolating polynomial for k points $y_n, y_{n-1}, \ldots, y_{n-k}$ (representing evenly-sampled points with $h = t_n - t_{n-1}$) can be written as

$$y(t) = y_n + \sum_{i=1}^{k} \left[\frac{1}{i!} \left(\prod_{j=0}^{i} (t - t_{n-j}) \right) \nabla^i y_n \right]$$

where $\nabla^i y_n$ is the approximation to the ith derivative at y_n using backwards differences.