CS 101.3: Numerical Geometric Integration

Homework Assignment #4
Due date: Feb 18th 2009 at the beginning of class.
All code should be submitted by email.
Note: This due date is a Wednesday since there is no class on Monday.
However, you will have another set due the following Monday, so start early!

Abstract

In this assignment you will be looking at the methods of dealing with constraints discussed in class, along with showing the equivalence of the Lagrangian and Hamitlon-Pontryagin points of view. Please note that the honor code applies: do the derivations yourself. If you have questions email patrickm@cs.caltech.edu - I will be happy to meet upon request.

1 Theory Part

1.1 Constrained Symplectic Euler

We showed in class that the map $(p_n, q_n) \to (\tilde{p}_{n+1}, q_{n+1})$ given by the first part of the constrained symplectic Euler method is symplectic on the constraint manifold. Show that the map $(\tilde{p}_{n+1}, q_{n+1}) \to (p_{n+1}, q_{n+1})$ given by the projection of $\tilde{p}_{n+1}$ onto the cotangent space of the constraint manifold is also a symplectic map, and hence the entire method $(p_n, q_n) \to (p_{n+1}, q_{n+1})$ is symplectic.

1.2 Bar-On-A-Circle

For the system explained in the implementation section, use an augmented Lagrangian to derive a constrained variational integrator for the system. Use a midpoint discretization of the potential energy.

1.3 Hamilton-Pontryagin and DEL equations

Show through substitution that the update rule obtained through the discrete Hamilton-Pontryagin setup (using $L(q, v)$) is the same as the standard DEL equations from the normal discrete Lagrangian (using $L(q_k, q_{k+1})$). You may use a midpoint discretization of the potential energy.

2 Implementation part

2.1 Bar-On-A-Circle Implementation

You will implement the following system: Two point masses q_1 and q_2 in $\mathbb{R}^2$ with masses m_1 and m_2 are attached to the ends of a massless bar of length 1. The midpoint of this bar is attached to and allowed to slide freely along the unit circle, and the only external force is gravity. For the simulations take $m_1 \neq m_2$ and initial conditions such that the behavior is interesting (for example, start the bar horizontal somewhere near the top of the circle). Make the code such that the initial conditions are easy to change (and comment where this is if it's not obvious).

1. Implement this system using explicit Euler and the standard projection method. Make an animation of the motion over time and plot the energy of the system over time.
2. Do the same using the symmetric projection method discussed in class along with the implicit midpoint integrator.
3. Do the same using your constrained variational integrator you derived above. For this one only also plot the magnitude of the constraint forces over time (plot both forces on top of each other on the same plot).
4. Do the same using a constrained variational integrator, only with $m_1 = 3m_2$ and with the bar attached to the circle at a point $2/3$ the way towards p_1 (i.e. the distance of q_2 from the unit circle should be twice that of q_1).