Online Geometric Optimization in the Bandit Setting Against an Adaptive Adversary

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Overview

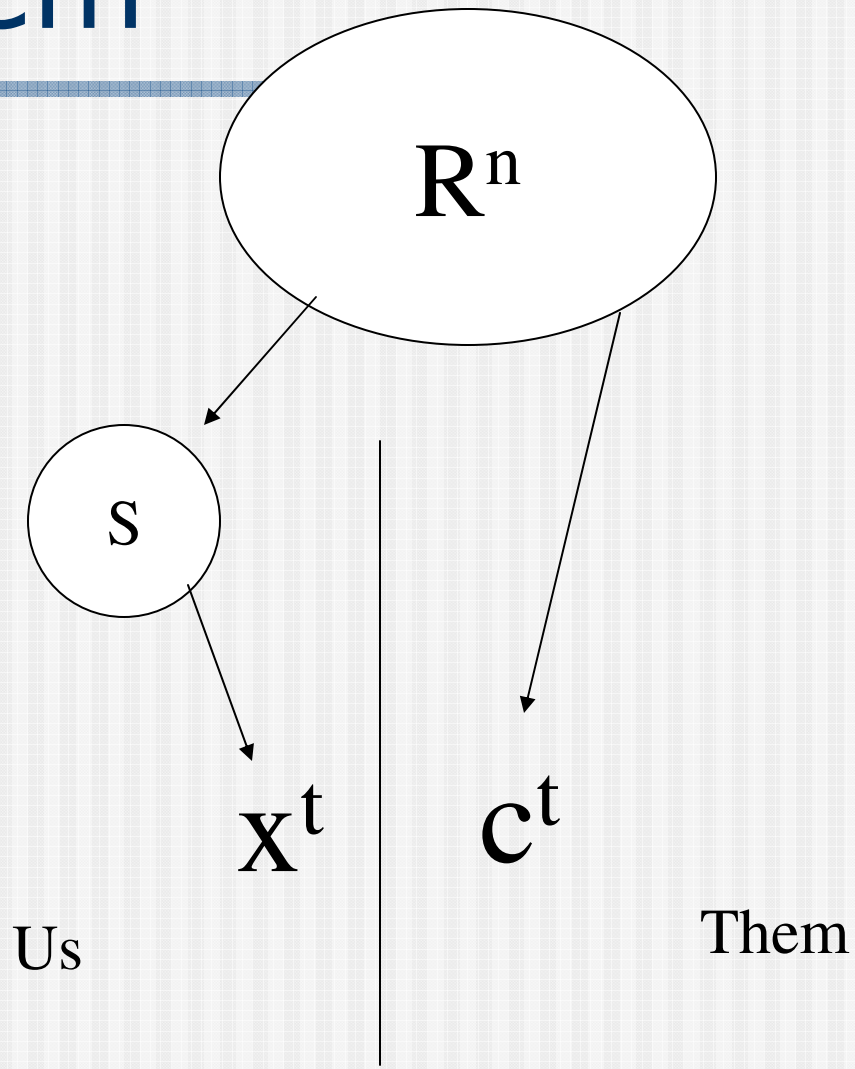
- This is a solution to the Bandit version of the previous problem (Kalai and Vempala's online decision problems).

Overview

- "Bandit Version" of problem
- Algorithm to solve Bandit Version
- Algorithm analysis
- Improvement?

Overall Problem

- S is bounded
- Cost vectors are bounded
- We choose x vector
- Opponent chooses c vector



Notation: superscripts denote iterations of the algorithm

Bandit Version

- What does this mean?
- At each step, we observe the total cost ($x^t \cdot c^t$), not the cost vector (c^t).
- The standard bandit setting can be found by specifying S appropriately.

Intuition

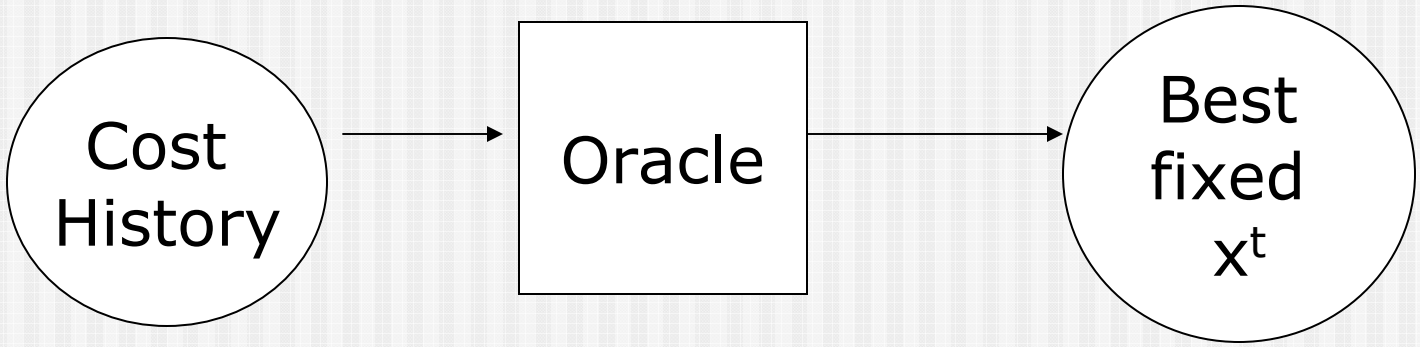
- Even a very smart opponent can't do anything worse to us than what he's already been doing (in the long run).
- So, if we choose x based on which static x would work the best for us in the past, it should work well for us for the next iteration as well.

Algorithm

- Need to minimize regret in the long term.
- Regret is the difference between our choice and the best choice.

Algorithm

- Begin by forming an "oracle"



For our purposes, we will use GEX (Geometric Experts Algorithm), based on the FPL algorithm from Kalai and Vempala.

Oracle, cont.

- In our case, the oracle takes in the cost history:
  - $\hat{c}^1$, $\hat{c}^2$, …, $\hat{c}^{t-1}$
- And outputs a distribution:
  - (choice 1:1%), (choice 2:3%), (choice 3:2%) … over all (infinite) choices in S
- Remember the intuition!

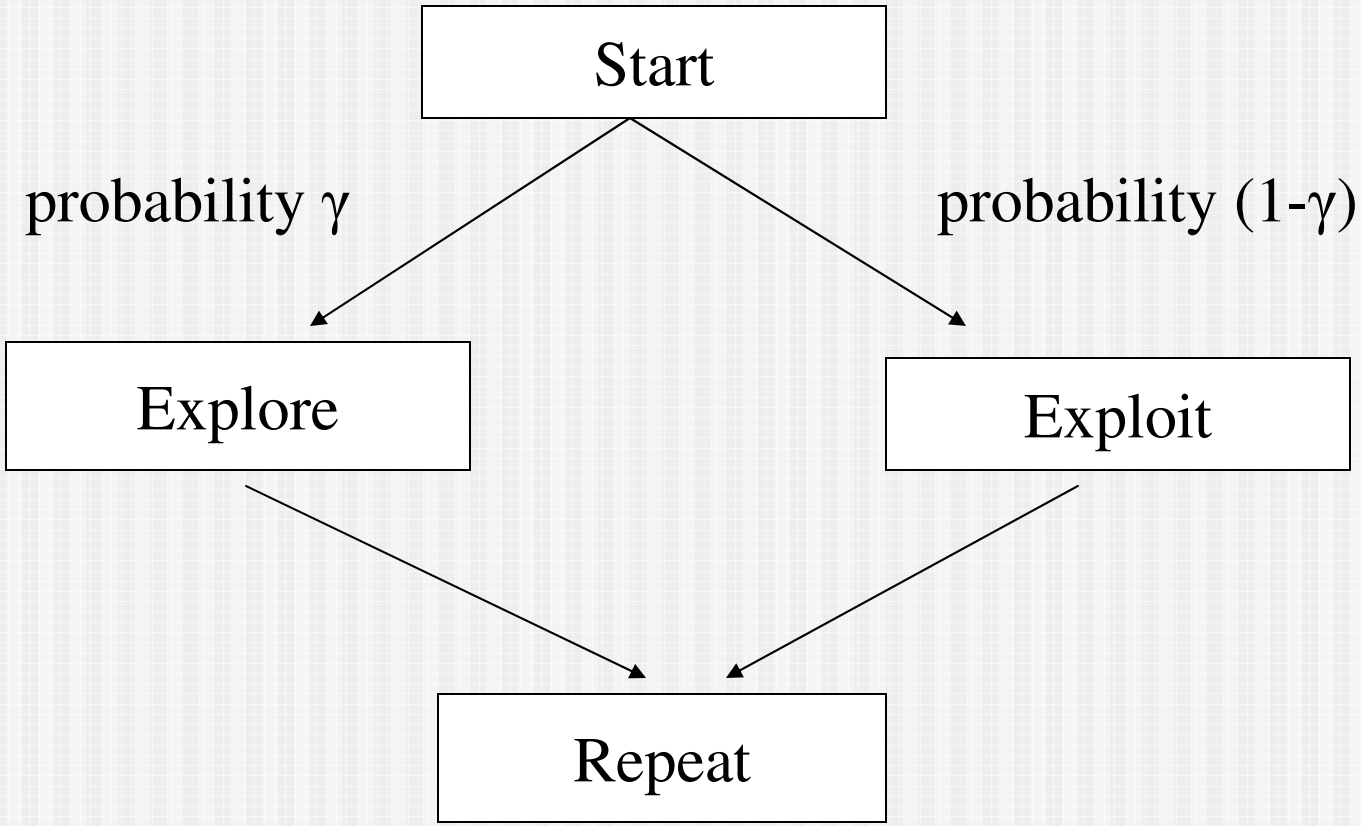
Oracle, cont.

- This distribution represents the best choice for x, considering all previous cost vectors.
- In other words, if we had to choose one x for all previous cost vectors, this would be it.
- It’s probabilistic.

Basis

- We also need a basis to span S.
- We can use an n by n matrix B.
- Each column in B is an element of S.

Algorithm


Start
probability γ
probability (1-γ)
Explore
Exploit
Repeat

Explore

- Randomly choose one of the basis vectors in B.
- Incur cost
- Store information to cost history.

Exploit

- Select x^t from the most current distribution given by the oracle GEX
- Incur cost $x^t \cdot c^t$
- $\hat{c}^t = (0,0,\ldots 0)$ (n-tuple) : Store a "zero" in the cost history to avoid biasing the history.
- Some information is lost here.

Return

- After each iteration, we need to update some data.
- The cost history gains either a zero (for Exploit) or a cost vector (for Explore).

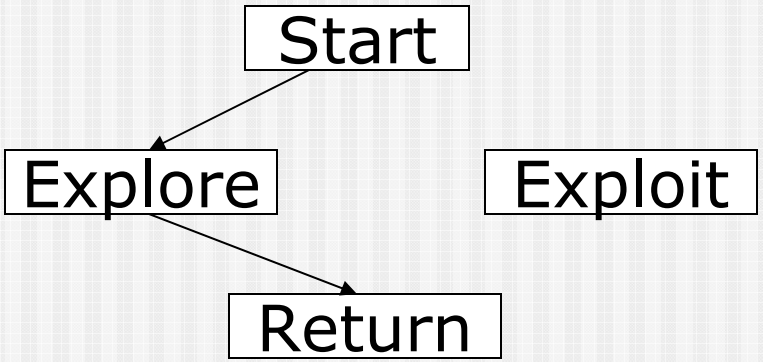
Example

- Consider n = 2 and
- Cost vector sum <= 2 (for bounding constraint) and
- S is [(-1,-1),(1,1)].

- Possible basis is (0,1) and (1,0).

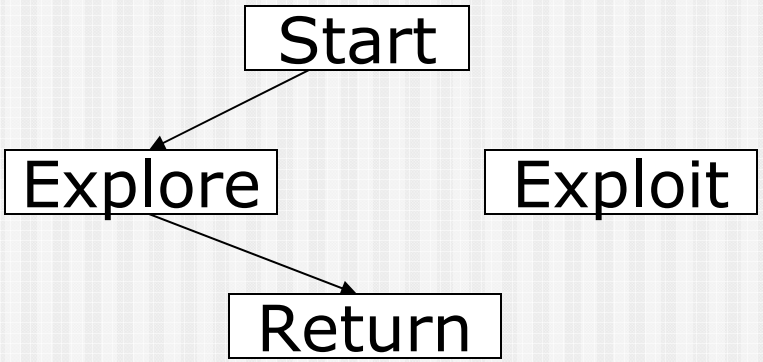
Example

- First iteration:
  - Biased coin flip chooses Explore:
  - Random selection chooses (0,1):
  - Opponent chooses (1,1) for costs:
  - Algorithm sees cost of 1.



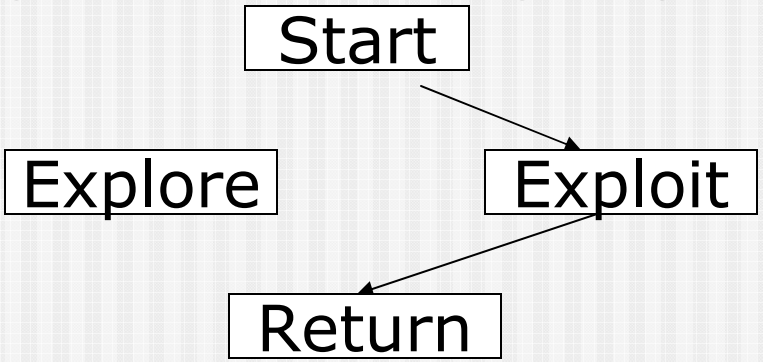
Example

- Second iteration:
  - Biased coin flip chooses Explore:
  - Random selection chooses (1,0):
  - Opponent chooses (1,1) for costs:
  - Algorithm sees cost of 1.



Example

- Third iteration:
  - Biased coin flip chooses Exploit:
  - Oracle returns random value from distribution (.3, .6)
  - Opponent chooses (1,1) for costs:
  - Algorithm sees cost of .9, stores 0



Example

- Continuing. . .
- In the long run:
  - Opponent chooses costs > 0, we choose x < 0.
  - Opponent chooses costs < 0, we choose x > 0.
- Opponent should choose cost of (0,0), which then minimizes our regret.

Algorithm Analysis

- Proof of $O(T^{3/4}\sqrt{(\ln T)})$ regret
- First, estimates.
- Our error in our estimates of the cost times a specific basis vector will decrease as we gain more information through exploration.

Inequalities

- E[loss(GEX)] <= E[optimal performance based on estimated c values] + terms
- This comes from analysis in another paper.

Inequalities

- E[loss(algorithm)] <= E[loss(GEX)]+terms
- This is because the algorithm is an improvement on the oracle itself, with the choice between exploring and exploiting.

Inequalities

- E[optimal performance based on estimated c values] <= E[optimal performance] + terms
- This actually only holds probabilistically, but it’s good enough for the proof.

Proof

- If we combine these inequalities and cleverly set γ equal to T^(-1/4), we get:
- E[regret(algorithm)]= $O(T^{3/4}\sqrt{(\ln T)})$

- Note that γ, instead of changing with time, is simply coupled to T.

The Point

- As we gain more information by exploring,
- we get better estimates of what the adaptive opponent will do
- which decreases our expected regret,
- which ends up being bounded.

Improvement

- This algorithm has $O(T^{3/4}\sqrt{(\ln T)})$
- For an oblivious adversary, $O(T^{2/3})$ is possible.
- For flat bandit problems, $O(\sqrt{T})$ is possible.
- Can this algorithm be improved? Maybe.
- Possibly the information from the exploit step can be better used.

Questions?