CS 153 Current topics in theoretical computer science	**Spring 2018**

Problem Set 1

Out: May 1 — *Due:* **May 11**

You are encouraged to work in groups of two or three; however you must turn in your own write-up and note with whom you worked. Please don't consult online solutions or original research papers or surveys containing such solutions while doing this problem set. Please attempt all problems.

1. Show that if $f : X \times Y \to \{0,1\}$ has a fooling set of size t, then $R_0^{\text{pub}}(f) \geq \log t$. Give an example of a function exhibiting an exponential gap between R_0^{pub} and $R_{1/3}^{\text{pub}}$.

2. This problem concerns the function TRIBES : $\{0,1\}^n \times \{0,1\}^n \to \{0,1\}$. We view Alice's input x as a $\sqrt{n} \times \sqrt{n}$ matrix and Bob's input y in the same way. Then

$$\text{TRIBES}(x,y) = \bigwedge_i \bigvee_j (x_{i,j} \wedge y_{i,j}).$$

   (a) Prove that $D(\text{TRIBES}) = \Theta(n)$.

   (b) Prove that $N^1(\text{TRIBES}) = \Theta(\sqrt{n}\log n)$. Hint: for the lower bound, find a 1-fooling set of size $\sqrt{n}^{\sqrt{n}}$.

   (c) Prove that $N^0(\text{TRIBES}) = \Theta(\sqrt{n})$.

3. Recall that in the CIS_G problem, Alice holds a clique in an n node graph G and Bob holds an independent set in G. Recall that we proved an upper bound $D(\text{CIS}_G) \leq O(\log^2 n)$.

   (a) Prove the lower bound $D(\text{CIS}_G) \geq \Omega(\log n)$, for some G.

   (b) Suppose that we discover a better upper bound $D(\text{CIS}_G) \leq O(\log^c n)$ (that holds for all G). Prove that in this scenario $D(f) = O(\log^c C^D(f))$, where $C^D(f)$ is the size of the smallest disjoint cover of f by monochromatic rectangles.