CS 151 Complexity Theory **Spring 2004**

Problem Set 1

Out: April 1 *Due:* **April 8**

Reminder: you are encouraged to work in groups of two or three; however you must turn in your own write-up and note with whom you worked. You may consult the course notes and the text (Papadimitriou). Please attempt all problems.

1. Function and decision problems. Given a function $f : \Sigma^* \to \Sigma^*$ with $|f(x)|$ polynomial in $|x|$, describe a *related* language L_f for which:

   $$L_f \in \mathbf{P} \quad \Leftrightarrow \quad f \text{ is computable in polynomial time.}$$

   This justifies our focus on decision problems rather than the more general notion of function problems.

2. Downward self-reducibility. For a language A, define

   $$A_{<n} = \{x \in A : \; |x| < n\}.$$

   Language A is said to be *downward self-reducible* if it is possible to determine in polynomial time if $x \in A$ using the results of polynomially-many queries of the form "$y \in A_{<|x|}$?" Show that every downward self-reducible language is in **PSPACE**.

3. Show that one of the following inequalities must hold: $\mathbf{L} \neq \mathbf{P}$ or $\mathbf{P} \neq \mathbf{PSPACE}$. Note that both are believed to be true, and no one knows how to prove either one is true.

4. Show that logspace reductions are closed under composition. Use this to prove that if language A is $\mathbf{P}$-complete, then $A \in \mathbf{L}$ implies $\mathbf{L} = \mathbf{P}$.

5. Use a padding argument to show that if $\mathbf{L} = \mathbf{P}$ then $\mathbf{PSPACE} = \mathbf{EXP}$.