Computational Constraints on Scientific Theories: Insights from Quantum Computation

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Feynman '82: "It has not yet become obvious
to me that there's no real
problem (with quantum mechanics)."
...

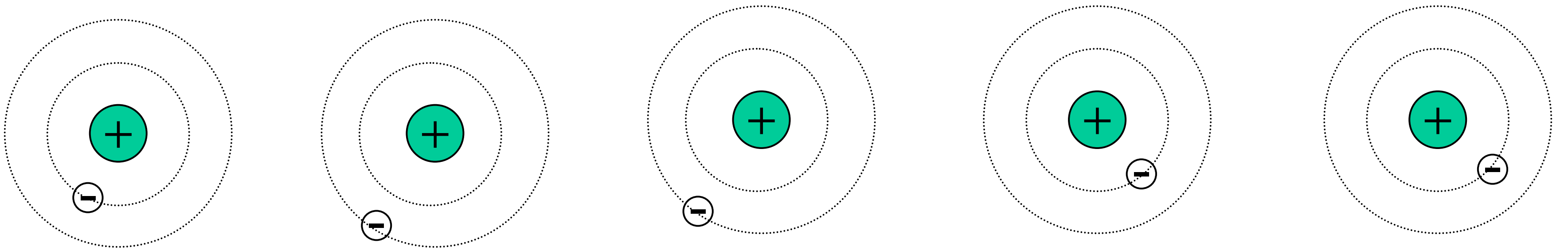
"Can I learn anything by asking this
question about computers ..."

What have we learned?

Nature appears to expend extravagant resources in:

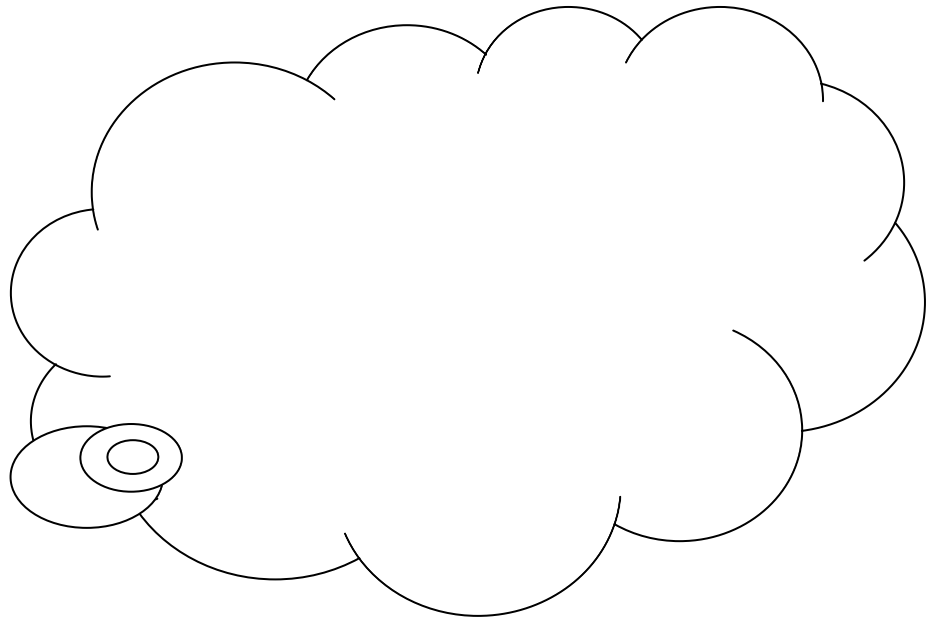
- Storing the state of a quantum system
- Evolving the state in time

Exponentially Large Hilbert Space



Exponentially Large Hilbert Space

Storing the state

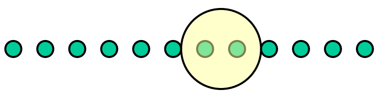


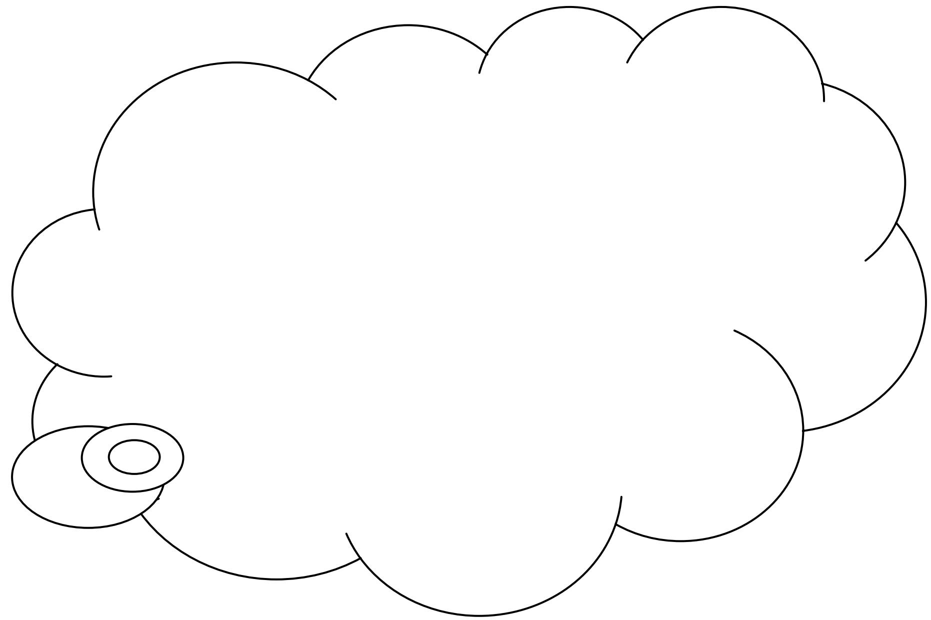
$$\Psi = \sum_x \alpha_x |x\rangle \qquad \sum_x |\alpha_x|^2 = 1$$

all n-bit strings

Quantum entanglement

Evolving the state





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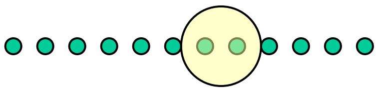
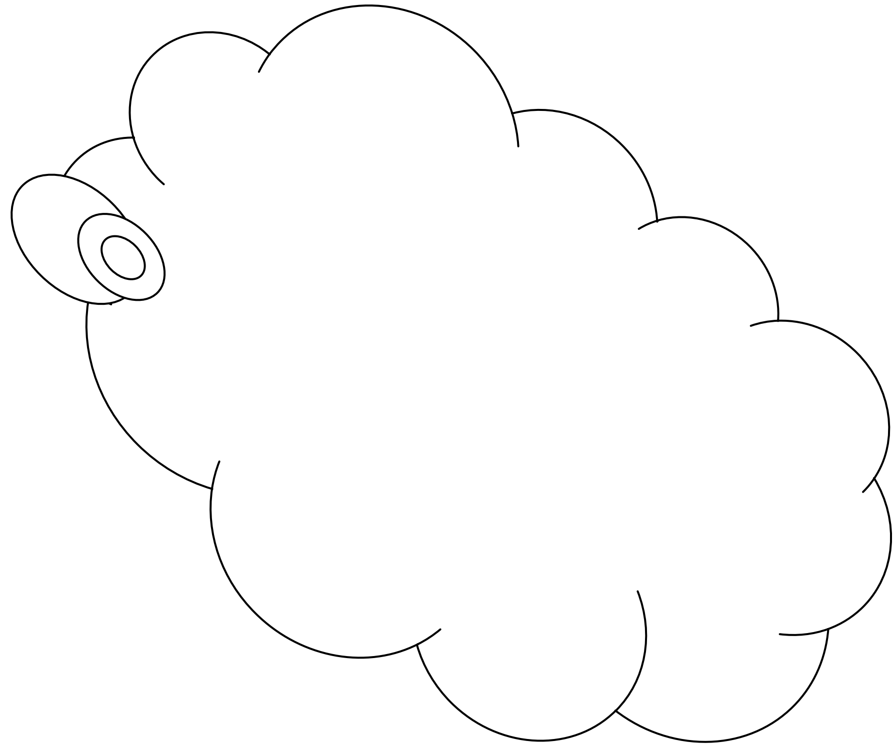
Evolving the state





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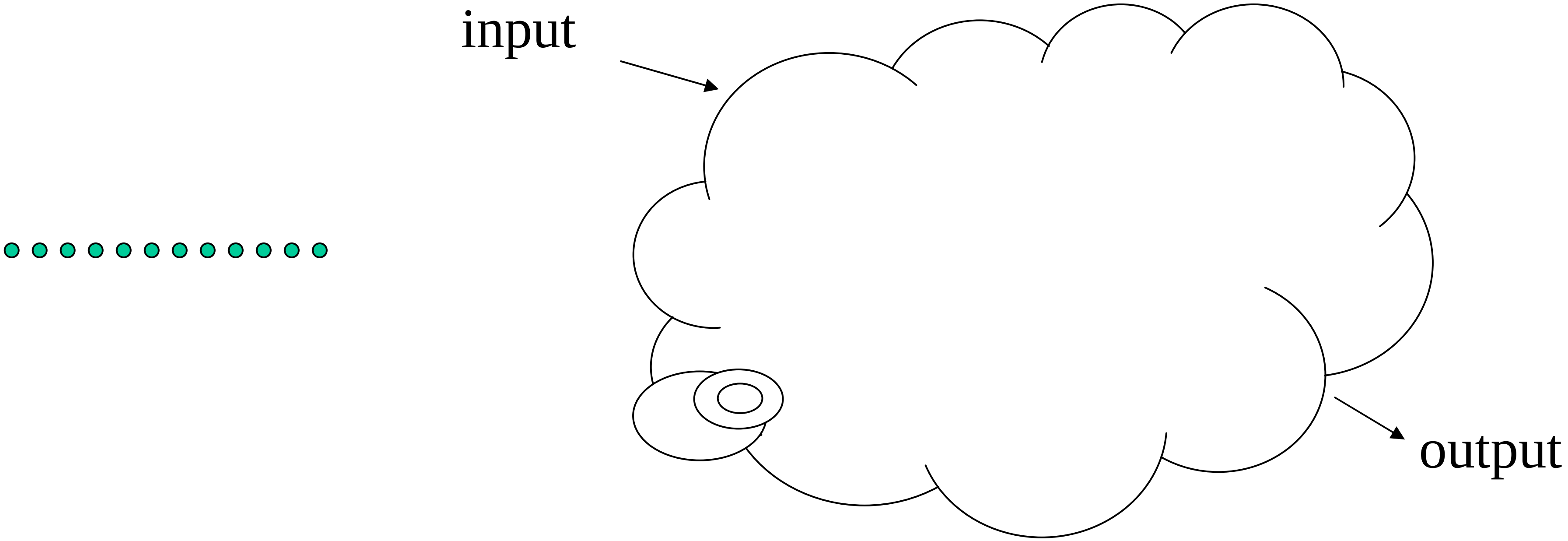
all n-bit strings

Limited Access - Measurement



$$\Psi = \sum_x \alpha_x |x\rangle \qquad \sum_x |\alpha_x|^2 = 1$$

- Measurement: See $|x\rangle$ with probability $|\alpha_x|^2$

Impact on Computer Science

- Rethink the foundations of computational complexity theory
- Quantum computers break modern cryptography
  - factoring, discrete log, …

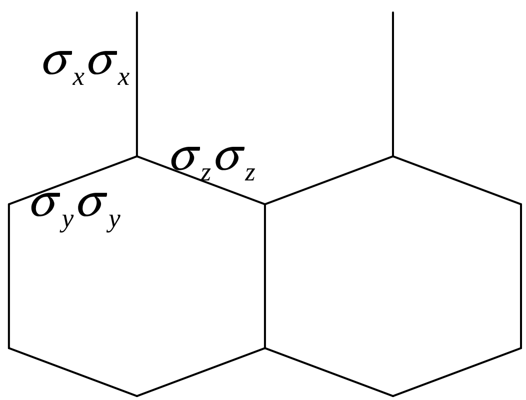
Implications of exponential resources in physics?

- QED – light and electrons
- Structure of atoms, chemical properties
- Novel large scale quantum phenomena
  - Bose-Einstein condensates
  - Lasers

Exponential resources?

- QED – single particle
- Atoms/Molecules - single particle + mean field theories …
- Bose-Einstein condensates – effectively two-state systems
- Bell states – 2 particle entanglement

- Topological quantum computing
- Fractional Hall effect
- Kitaev's Honeycomb lattice:



Exact solution. Ground state highly entangled
fermionic operators

Statistical Properties:

God does not play dice with the universe --- Einstein

Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the Old One. **I, at any rate, am convinced that He does not throw dice.**

---letter to Max Born 1926.

Statistical Properties:

God does not play dice with the universe --- Einstein

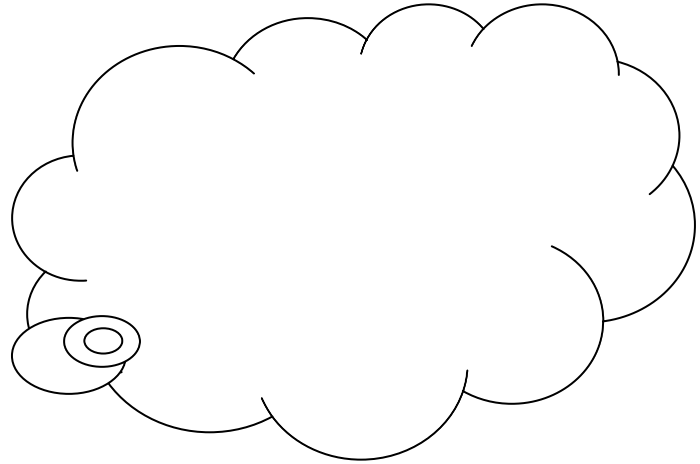
Bell inequality violations demonstrate that God does play dice…

Computational resources:

- The Old One does not use exponential resources

- Occam's razor

………..



- Falsifiability

The criterion of *the scientific status of a theory is its* ***falsifiability****, or refutability, or testability*.

Some theories are more testable, more exposed to refutation, than others; they take, as it were, greater risks.

--Karl Popper

Is Quantum Physics Falsifiable?

- Single particle quantum physics has been verified to exquisite accuracy.
- Multi-particle quantum systems – exponentially hard to compute what the theory predicts.
- What about predictions using mean field approximations/perturbation theory?
- Can any theory that requires exponential resources possibly be refuted?

Computational Complexity Theory

One-way functions:

- $y = f(x)$ can be efficiently computed on input x.

- f is hard to invert: given y, hard to recover $x = f^{-1}(y)$

e.g. Factoring $N = pq$

Shor's quantum factoring algorithm.

NMR QC: 15=3x5

[Chuang, et al]

[Braunstein, Caves, Jozsa, Linden, Popescu, Schack]

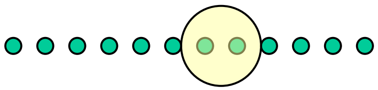
State of quantum computer is separable mixed state.

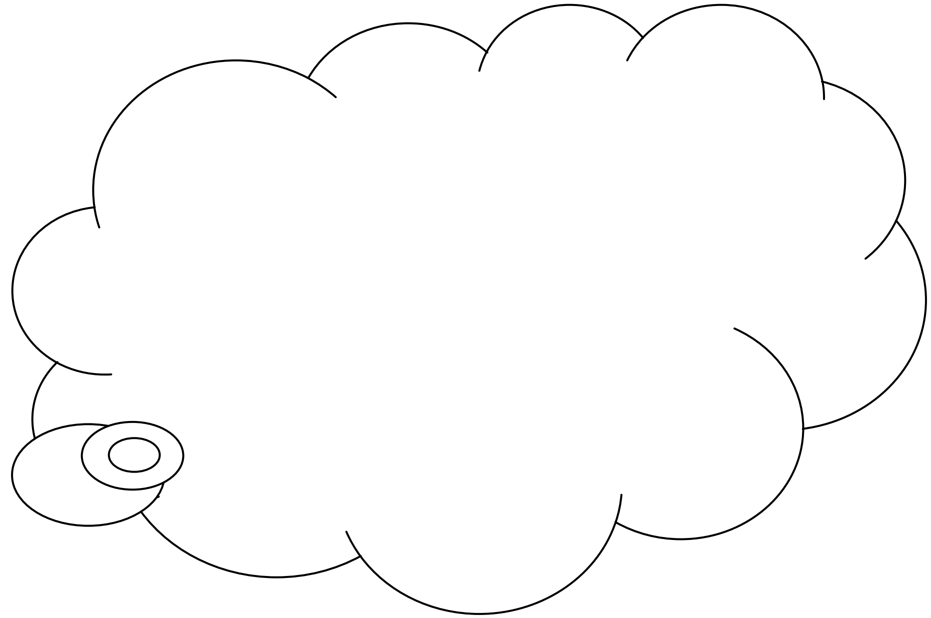
Mixture: $\left|\varphi_1\right\rangle \otimes \ldots \otimes \left|\varphi_n\right\rangle$ *with probability* p

After application of quantum gate mixture looks entangled, but can be written as equivalent separable mixed state.

Open: Can we perform non-trivial quantum computation in this model.

Infinite Precision?





$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\Psi = \sum_x \alpha_x |x\rangle$$

all n-bit strings

Error: $\theta \to \theta \pm \varepsilon$

$$U = R_\theta \otimes I$$

Quantum State Tomography
PAC model

- Unknown n-qubit quantum state $|\varphi\rangle$
- Can repeatedly prepare $|\varphi\rangle$
- Wish to learn the state.

Problem: Exponential number of parameters to "know" the state.

What can one do?

Pretty Good Tomography

[Aaronson '06] Inspired by computational learning theory Valiant's PAC model.

Setting: Assume experimenter has certain (possibly very large number of) measurements she cares about – possibly to varying degrees. Each time she selects a measurement from a distribution D that reflects their importance.

Want: After m experiments want to predict the results of future experiments almost as well as if quantum state completely known.

Pretty Good Tomography

Unknown n-qubit quantum state $|\varphi\rangle$

Distribution D on possible measurements.

Get to see m samples

Must learn $|\varphi\rangle$ sufficiently well to predict outcome of measurement from D with probability at least 1-e.

O(n/poly(e)) samples suffice.

Quantum Random Access Codes

[Ambainis, Nayak, Ta-Shma, Vazirani]

Disposable Quantum Phonebook:

$d = 10^6$ phone numbers

Wish to store them using $n \ll d$ quantum bits:

Can look up any phone number of your choice

Measurement disturbs system, so must discard phonebook.

Theorem: $d = O(n)$.

Key Ideas

- Assume for simplicity 2 outcome measurements.
  - wish to know whether outcome 1 more likely.
- Fix any m measurements. Max number of distinct behaviors?

	M_1	M_2	M_3	$\cdots$	M_m
$\|\phi_1\rangle$	0	1	1		1
$\|\phi_2\rangle$	1	0	0		0
$\|\phi_3\rangle$	0	1	0		1
		$\vdots$			
$\|\phi_k\rangle$	1	1	1		1

Key Ideas

- CLT: number of behaviors is either 2^m or m^d
- Number of samples to reconstruct O(d)
- (n,d) random access code implies d = O(n).

	M_1	M_2	M_3	$\cdots$	M_m
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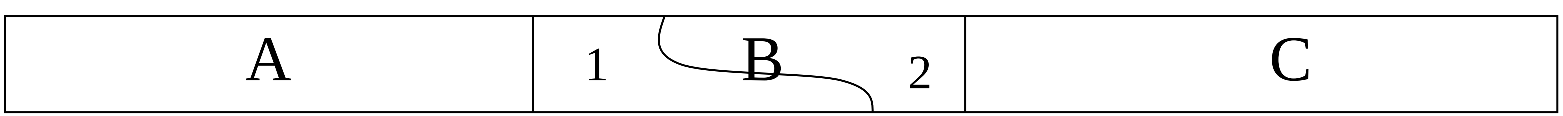
Classical Simulation of Quantum Systems

- Quantum entanglement necessary for quantum computation.
- Systems with low entanglement can be efficiently simulated
- Succinct description

Vidal Polynomial time simulation of one dimensional spin chains with O(log n) entanglement length.

$$\rho_{AC} = \rho_A \otimes \rho_C$$



$$\rho_{ABC} = \rho_{AB1} \otimes \rho_{B2C}$$

Challenges

- Are there special classes of quantum states for which the learning algorithm is also efficient in time complexity.

- For special classes, can we actually learn the quantum state, not just a predictor for measurements.

Conclusions

- Exponential Hilbert space – challenge + opportunity.
- Quantum algorithms provide a falsifiable consequence of multi-particle quantum physics.
- Learning theory for quantum states and efficiently simulatable quantum systems.
- Efficient classical simulation of special quantum systems.