Inapproximability for VCG-Based Combinatorial Auctions

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Auctions

We look at combinatorial auctions with n bidders and m items.

Each bidder submits his valuation function, then the mechanism assigns items and payments to bidders.

We look at the relatively simple case of budget-additive valuations, with the goal of maximizing the social welfare.

Budget-Additive

A budget-additive bidder i has a value v_{ij} for each item j, as well as a budget b_i. The total value of a set S is

$$v_i(S) = \min\left(\sum_{j \in S} v_{ij}, b_i\right)$$

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This budget is not to be confused with a spending limit. Rather, the bidder stops desiring more after a value of b_i is reached.

Budget-Additive Example

Suppose I value video games by hours played per week

Budget-Additive Example

Value: 40

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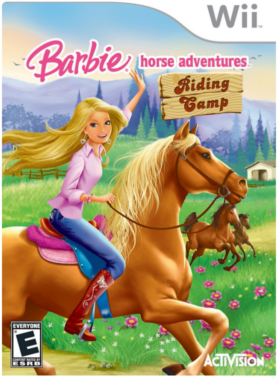
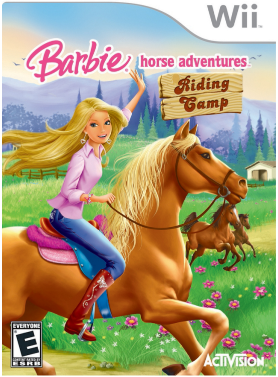


Value: 60

Budget-Additive Example

Value: 40



Value: 60

Value: 80

Why Budget-Additive?

- Budget additive auctions are simple, but NP-hard
- A special case of many commonly studied valuation classes
  - subadditive
  - submodular
  - XOS
- So a hardness result for budget-additive auctions is very strong

How do we measure allocations?

Definition (Social Welfare)

The social welfare of an allocation $S_1, \ldots, S_n$ with valuation functions $v_1, \ldots, v_n$ is

$$\sum_i v_i(S_i)$$

The VCG Mechanism

- By participating in the auction, each bidder harms the others

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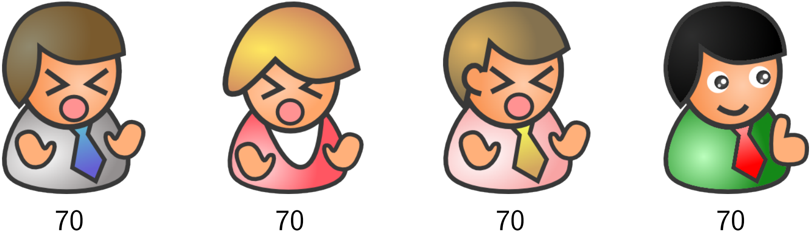

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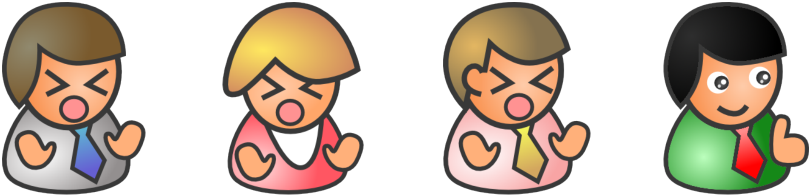

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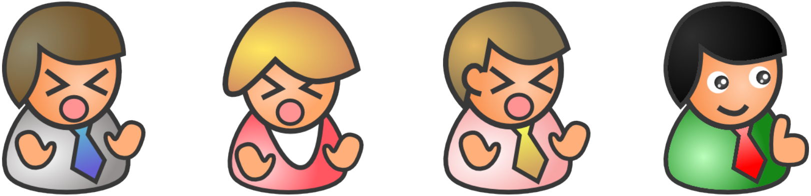

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- To balance incentives, each player is charged for this harm
- Intuitively, the player wants the social welfare maximized
- This only works if the mechanism is maximal-in-range (MIR)

Maximal-In-Range

- An allocation function maps bids to allocations of items
- Each allocation function f has a range R
- f is maximal-in-range if it maximizes over R

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Example

Grouping all items into one lot, we can maximize over a range of size n. This yields a $1/n$ approximation.

Why focus on VCG/Maximal-In-Range?

- MIR mechanisms are the only known general truthful class of auction mechanisms
- The best known truthful mechanism for subadditive auctions is MIR

Why so many authors?

This work is the combined results of 3 groups of authors:

- Mossel, Papadimitriou, Schapira, Singer
- Buchfuhrer, Umans
- Dughmi, Fu, Kleinberg

Mossel, Papadimitriou, Schapira, Singer

The original insight: If a maximal-in-range mechanism M has a range with large VC dimension, it is implicitly solving a smaller version of the problem [Papadimitriou, Schapira, Singer, 2008]

If the sub-auction is polynomially large, M implies $NP \subseteq P/poly$

Showed that with a constant number n of bidders, an approximation of $\frac{n+1}{2n}$ implies a polynomial VC dimension

Buchfuhrer, Umans

Used combinatorial techniques which more closely fit the problem and relaxed the notion of VC dimension to show a hardness to approximate within

$$\max\left((1+\epsilon)/n, 1/m^{1/2-\epsilon}\right)$$

This essentially matches a known MIR mechanism that achieves

$$\max\left(1/n, 1/O(\sqrt{m})\right)$$